COMPTES RENDUS
(DOKLADY)
DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1939

IMPERIAL MYCOLOGICAL
22 JULY 1939
INSTITUTE

VOLUME XXIII

№ 5

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

AVIS AUX AUTEURS

Les «Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS» paraissent tous les dix jours, neuf numéros formant un volume d'environ 850 pages pourvu d'une table des matières.

Les «Comptes Rendus de l'Académie des Sciences de l'URSS» publient des communications courtes précisant l'essentiel des travaux plus volumineux en préparation. Ces articles, ne dépassant pas comme règle dix mille signes typographiques, sont donnés en l'une des trois langues: français, anglais, allemand. Ils embrassent les sciences mathématiques, physiques, naturelles et techniques, reproduisant les résultats de recherches scientifiques actuellement en cours ou à peine achevées qui représentent les plus récentes données de l'investigation scientifique.

Les auteurs ont droit chacun à 25 extraits gratuits.

Les manuscrits doivent être adressés à la Rédaction des «Comptes Rendus», Moscou 19, Volchonka, 14, chambre 211.

15 Mai

COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

COMITÉ DE RÉDACTION

A. Borissiak, de l'Académie, A. Fersman, de l'Académie, A. Frumkin, de l'Académie,
A. Richter, de l'Académie, S. Wawilow, de l'Académie

NOUVELLE SÈRIE

7-me année

Paraissant tous les dix jours

1939

VOLUME XXIII, № 5

TABLE DES MATIÈRES

Pages

MATHÉMATIQUES

S. A. Tchounikhin. Über das Zentrum der Sylowuntergruppen der einfachen Gruppen 415
A. Youdine. Solution des deux problèmes de la théorie des espaces semi-ordonnés 418
M. Frank. Approximation des polyèdres simplement connexes au moyen de surfaces rationnelles 423
J. Geronimus. On the Orthogonality of a System of Polynomials on Several Contours 427
I. Gelfand. On Normed Rings 430
B. Vulich. Sur la métrisation de convergences dans les espaces linéaires 433
P. Novikoff. Sur quelques théorèmes d'existence 438

MÉCANIQUE

P. Riz. On the Deformations of Naturally Twisted Bars 441

PHYSIQUE STELLAIRE

A. I. Lebedinsky. Equation of the Transport of Energy by Convection Currents in Stars 445

PHYSIQUE

P. Krasnooshkin and E. Pumper. Supersonic Absorption in Helium . 448

V. Berestecki. Shape of the Beta-Spectrum in the Case of the Forbidden Transitions 450
J. S. Shur. Heat Treatment of Ferromagnetic Polycrystals in a Magnetic Field 455
B. Klarfeld and **I. Poletaev.** Pressure Gradient in the Positive Column 460
B. Klarfeld and **I. Poletaev.** Gas Rarefaction at Constrictions in the Positive Column 465

CHIMIE

A. P. Barkhash. Anaerobic Breakdown of Phosphogluconic Acid in the Presence of Acetaldehyde 467

MINÉRALOGIE

J. D. Gotman. Some Anomalies in the Properties of Cassiterite 470

GÉNÉTIQUE

V. V. Svetozarova. Second Genom of *Triticum Timopheevi* Zhuk. . . . 473
D. R. Gabe. Inheritance of Sex in *Mercurialis annua* in Relation to Cytoplasmatic Theory of Sex-Inheritance 478
N. K. Koltzoff, Corresponding Member of the Academy. On the Methods of Artificially Inducing Polyploids by Treatment with Colchicine . . 482

PHYSIOLOGIE VÉGÉTALE

I. N. Holubinsky and **M. S. Schloss.** A Contribution to the Problem of Parthenogenesis in *Humulus Lupulus* L. 486

ANATOMIE DES PLANTES

I. I. Chkhubianishvili. Histological Investigation of Leaf of *Primula nivalis* Pall. and *Cerastium ovatum* Boiss. 488
F. Kh. Jager and **M. S. Jakovlev.** Condition ol Starch in Potato at Different Stages of Tuber Ripening 491

ZOOLOGIE

S. E. Kleinenberg. On the Methods of Estimating the Stocks of the Black Sea Dolphin *(Dolphinus delphis ponticus* Barabasch*)* 495

MORPHOLOGIE EXPÉRIMENTALE

K. Z. Kan. Morphological Response of the Transplanted Hypophysis to Castration and Thyroidectomy 499

MICROBIOLOGIE

L. A. Ivanov, Corresponding Member of the Academy, and **E. A. Stern.** Methodics of Breathing Determinations in Microbes 503

PALÉOZOOLOGIE

B. B. Rohdendorf. A New *Protelytropteron* from the Permian of the Urals 506

Rédacteur en chef A. Fersman. Rédacteur S. Bernstein.

En composition 11/V 1939. En impression 15/VI 1939. Nombre de feuilles d'impression 6. Nombre de caractères par feuille d'impression 65.000. Pages 413—508. Format du papier 72×110 cm. Tirage 1000. Glavlit A 5928. Numéro d'édition 2070. Numéro typographique 1032.

Imprimerie № 16, Moscou, Trekhproudnij péréoulok, 9.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

MATHEMATIK

ÜBER DAS ZENTRUM DER SYLOWUNTERGRUPPEN DER EINFACHEN GRUPPEN

Von S. A. TCHOUNIKHIN

(Vorgelegt von O. J. Schmidt, Mitglied der Akademie, d. 17. III. 1939)

1. Es gilt der folgende, von mir bewiesene

Satz I (1). *Sei $\mathfrak{H}$ eine Untergruppe vom Index k der Gruppe $\mathfrak{G}$ und sei $\mathfrak{K}$ eine Untergruppe von $\mathfrak{H}$, die die Kommutatorgruppe von $\mathfrak{H}$ umfasst. Sei in $\mathfrak{H}$, aber nicht in $\mathfrak{K}$, ein Element A enthalten mit der Eigenschaft, dass für jedes λ alle Elemente von $\mathfrak{H}$, die mit A^λ in $\mathfrak{G}$ konjugiert sind, dem System $\mathfrak{K}A^\lambda$ angehören, und sei A^σ die kleinste Potenz von A, die in $\mathfrak{K}$ auftritt. Wenn k durch σ nicht teilbar ist, so ist die Gruppe $\mathfrak{G}$ nicht einfach.*

Wie ich (2) und L. Weisner (3) gezeigt haben, kann man diesen Satz I bei den Untersuchungen der einfachen Gruppen benutzen. Im folgenden wird eine weitere Anwendung dieses Satzes gegeben.

2. Sei $\mathfrak{P}$ eine der Sylowuntergruppen der Ordnung p^δ einer Gruppe $\mathfrak{G}$ der Ordnung $p^\delta n$. Es sei ferner ρ die kleinste Zahl, für welche $p^\rho - 1$ und n einen von 1 verschiedenen gemeinsamen Teiler haben. Die Gruppe $\mathfrak{C}$ der Ordnung p^η sei das Zentrum von $\mathfrak{P}$. Setzen wir voraus, dass die fundamentale Basis von $\mathfrak{C}$ nur die folgenden Elemente enthält:

die Menge (A_1) der n_1 Elementen von der Ordnung p^{α_1},
die Menge (A_2) der n_2 Elementen von der Ordnung p^{α_2},
. . . . . .
die Menge (A_s) der n_s Elementen von der Ordnung p^{α_s},

wobei $\alpha_1 > \alpha_2 > \alpha_3 > \ldots > \alpha_s$ und $\alpha_{s+1} = 0$ ist.

Wir führen die folgenden Bezeichnungen ein:

1) für $1 < i \leqslant s$, sei τ_i die kleinste der Zahlen $\alpha_{i-1} - \alpha_i$ und $\alpha_i - \alpha_{i+1}$;
2) für $i = 1$, sei $\tau_1 = \alpha_1 - \alpha_2$;
3) für $s = 1$, sei $\tau_1 = \alpha_1$.

Beweisen wir nun den folgenden

Hilfssatz. *Die Gruppe $\mathfrak{C}$ enthält stets für jedes τ_i ein Paar charakteristischer Untergruppen*

$$\mathfrak{A}_i \supset \mathfrak{B}_i$$

mit der Eigenschaft, dass die Faktorgruppe $\frac{\mathfrak{A}_i}{\mathfrak{B}_i}$ eine Abelsche Gruppe der Ordnung $p^{n_i \tau_i}$ und des Typus $(p^{\tau_i}, p^{\tau_i}, \ldots, p^{\tau_i})$ ist.

Der Beweis wird etwa wie folgt geführt [vgl. W. Burnside (4)].

Betrachten wir die folgenden drei charakteristischen Untergruppen $\mathfrak{M}$, $\mathfrak{N}$, $\mathfrak{S}$ von $\mathfrak{C}$. $\mathfrak{M}$ ist eine Untergruppe, die von allen Elementen von $\mathfrak{C}$, deren Ordnungen die Zahl p^{α_i} teilen, erzeugt wird; $\mathfrak{N}$ ist die von allen Elementen von $\mathfrak{C}$, deren Ordnungen die Zahl $p^{\alpha_i-\tau_i}$ teilen, erzeugte Untergruppe, und $\mathfrak{S}$ — die von den p^{τ_i} Potenzen aller Elemente der fundamentalen Basis von $\mathfrak{C}$ erzeugte Untergruppe.

Die fundamentale Basis von $\mathfrak{M}$ besteht aus den folgenden Elementen:

$$(A_j^{p^{\alpha_j-\alpha_i}}) \quad \text{für} \quad j=1, 2, 3, \ldots, i-1,$$

und

$$(A_j) \quad \text{für} \quad j=i,\ i+1, \ldots, s.$$

Die fundamentale Basis von $\mathfrak{N}$ besteht aus den folgenden Elementen:

$$(A_j^{p^{\alpha_j-\alpha_i+\tau_i}}) \quad \text{für} \quad j=1, 2, 3, \ldots, i,$$

und

$$(A_j) \quad \text{für} \quad j=i+1, i+2, \ldots, s.$$

Die fundamentale Basis von $\mathfrak{S}$ besteht aus den Elementen:

$$(A_j^{p^{\tau_i}}),\ i=1, 2, 3, \ldots, i, i+1, \ldots, s.$$

Daraus folgt, dass die fundamentale Basis der Gruppe $\mathfrak{S}\mathfrak{N}$ die folgende ist:

$$(A_j^{p^{\tau_i}}) \quad \text{für} \quad j \leqslant i$$

und

$$(A_j) \quad \text{für} \quad j > i.$$

Betrachten wir die fundamentale Basis des Durchschnittes $\mathfrak{D}$ der Gruppen $\mathfrak{M}$ und $\mathfrak{S}\mathfrak{N}$. Für $j < i$ haben wir offenbar $\alpha_j - \alpha_i \geqslant \alpha_{i+1} - \alpha_i \geqslant \tau_i$. Daraus folgt, dass die fundamentale Basis von $\mathfrak{D}$ aus folgenden Elementen besteht:

$$(A_j^{p^{\alpha_j-\alpha_i}}) \quad \text{für} \quad j<i, \ (A_i^{p^{\tau_i}})$$

und

$$(A_j) \quad \text{für} \quad j>i.$$

Da $\mathfrak{M}$, $\mathfrak{N}$ und $\mathfrak{S}$ charakteristische Untergruppen von $\mathfrak{C}$ sind, so ist $\mathfrak{D}$ auch eine charakteristische Untergruppe von $\mathfrak{C}$. Es ist klar, dass $\mathfrak{M} \supset \mathfrak{D}$ und die Faktorgruppe $\frac{\mathfrak{M}}{\mathfrak{D}}$ von der Ordnung $p^{n_i\tau_i}$ und dem Typus $(p^{\tau_i}, p^{\tau_i}, \ldots p^{\tau_i})$ ist.

3. Satz II. *Wenn $\mathfrak{G}$ eine einfache Gruppe ist, so folgt aus der Ungleichung $n_i < \rho$ eine andere: $\tau_i \leqslant \delta - \eta$.*

Beweis. Setzen wir voraus, dass für eine Zahl n_i die Ungleichung $n_i < \rho$ gilt. Betrachten wir die Gruppen $\mathfrak{A}_i$ und $\mathfrak{B}_i$ des Hilfssatzes. Sei $\mathfrak{A}_i = \mathfrak{B}_i + \mathfrak{B}_iB_2 + \ldots + \mathfrak{B}_iB_b$, $b = p^{n_i\tau_i}$, eine Zerlegung der Gruppe $\mathfrak{A}_i$ nach $\mathfrak{B}_i$. Sei B eines der Elemente $B_2, B_3, \ldots, B_b$.

Es ist bekannt (5), dass alle Elemente von $\mathfrak{C}$, die mit B^λ in $\mathfrak{G}$ konjugiert sind, bereits im Normalisator $\mathfrak{B}\mathfrak{P}$ von $\mathfrak{P}$ (in Bezug auf die ganze Gruppe $\mathfrak{G}$) konjugiert sind. Die Gruppe $\mathfrak{A}_i$, als eine charakteristische Untergruppe von $\mathfrak{C}$, ist in $\mathfrak{B}\mathfrak{P}$ invariant. Daraus folgt, dass alle Elemente

von $\mathfrak{C}$, die mit B^λ in $\mathfrak{G}$ konjugiert sind, sind in der Gruppe $\mathfrak{A}_i$ enthalten, d. h. aus der Gleichung

$$G^{-1}B^\lambda G \subset \mathfrak{C};\ G \subset \mathfrak{G}$$

folgt

$$G^{-1}B^\lambda G = C^{-1}B^\lambda C;\ C \subset \mathfrak{B}\mathfrak{P};\ C^{-1}B^\lambda C \subset \mathfrak{A}_i.$$

Transformieren wir alle Elemente von $\frac{\mathfrak{A}_i}{\mathfrak{B}_i}$ durch C und wir erhalten einen Automorphismus der Gruppe $\frac{\mathfrak{A}_i}{\mathfrak{B}_i}$. Die Ordnung von C und die Zahl

$$(p^{n_i}-1)(p^{n_i-1}-1)\ldots(p-1)$$

sind teilerfremd, weil $n_i < \rho$ ist. Daraus folgt, dass der durch C induzierte Automorphismus von $\frac{\mathfrak{A}_i}{\mathfrak{B}_i}$ ist der identische Automorphismus, d. h. es ist

$$C^{-1}\mathfrak{B}_i B^\lambda C = \mathfrak{B}_i B^\lambda,$$

oder

$$\mathfrak{B}_i C^{-1} B^\lambda C = \mathfrak{B}_i B^\lambda.$$

Daraus folgt, dass alle Elemente von $\mathfrak{C}$, die mit B^λ in $\mathfrak{G}$ konjugiert sind, dem System $\mathfrak{B}_i B^\lambda$ angehören. Nach dem Satz I (wenn man $\mathfrak{H} = \mathfrak{C}$ und $\mathfrak{K} = \mathfrak{B}_i$ setz), ist $\mathfrak{G}$ nicht einfach, wenn $\tau_i > \delta - \eta$ ist. Damit ist der Satz II bewiesen.

4. Wenn $\mathfrak{P}$ eine Abelsche Gruppe ist, so ist $\delta = \eta$ und die Ungleichung $\tau_i \leqslant \delta - \eta$ wird unmöglich (da $\tau_i > 0$ ist). Folglich ist $n_i \geqslant \rho$. Wir erhalten somit den folgenden

Satz III. *Sei $\mathfrak{P}$ der Ordnung p^δ eine Abelsche Sylowuntergruppe einer einfachen Gruppe $\mathfrak{G}$ der Ordnung $p^\delta n$. Sei ρ die kleinste Zahl, für welche $p^\rho - 1$ und n einen von 1 verschiedenen gemeinsamen Teiler haben. Dann ist die Anzahl von Elementen gleicher Ordnung in der fundamentalen Basis von $\mathfrak{P}$ mindestens gleich ρ.*

Aus dem Satz III folgt

Satz IV. *Sei $\mathfrak{P}$ wie in Satz* III. *Dann enthält $\mathfrak{P}$ nur solche Elemente, deren Ordnungen $\sqrt[\rho]{p^\delta}$ nicht übertreffen.*

5. Der Satz IV ist eine Verschärfung des folgenden von mir früher erhaltenen Satzes (6):

Sei $\mathfrak{G}$ eine einfache Gruppe der Ordnung $p^\delta n$ (p ist eine Primzahl, $p(p-1)$ und n sind teilerfremd). Sei $\mathfrak{P}$ eine der Untergruppen der Ordnung p^δ. Dann enthält das Zentrum der Gruppe $\mathfrak{P}$ nur solche Elemente, deren Ordnungen $\sqrt{p^\delta}$ nicht übertreffen.

Eingegangen
d. 23. III. 1939.

ZITIERTE LITERATUR

[1] Serge Tchounikhin, Mathem. Ann., **112**, 92 (1935); Rec. Math. de Moscou; **5** (47), No 3 (1939). [2] S. A. Tchounikhin, C. R. Acad. Sci. URSS, XX, № 2—3 (1938). [3] L. Weisner, Duke Math. Journ., **2**, № 4 (1936). [4] W. Burnside, Theory of groups of finite order, sec. ed., p. 108—109. [5] W. Burnside, loc. cit, p. 155. [6] Serge Tchounikhin, Mathem. Ann., **112**, 95 (1935).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

MATHÉMATIQUES

SOLUTION DES DEUX PROBLÈMES DE LA THÉORIE DES ESPACES SEMI-ORDONNÉS

Par A. YOUDINE

(Présenté par S. N. Bernstein, de l'Académie, le 25. III. 1939)

Cet article contient la solution des deux problèms posés par L. Kantorovitch. Les problèmes à démontrer sont: 1) tout espace semi-ordonné à un nombre fini de dimensions est isomorphe à l'espace Euclidien; 2) chaque relation qui a lieu identiquement pour les nombres réels reste vraie pour les éléments d'un espace semi-ordonné quelconque. Ces problèmes sont liés par la manière de les resoudre. Sauf les notations topologiques habituelles (1) dont nous nous servons, nous désignerons par: $L(M)$ l'ensemble linéaire minimum contenant l'ensemble M; $[M]$ — l'enveloppe convexe; $M+(y)$ — l'ensemble qu'on obtient en ajoutant l'élément y à M; $M+y=\underset{x}{\mathcal{E}}(x-y\in M)$; $(x\rightarrow y)$ — le rayon qui part de x vers y.

Nous nous occuperons des espaces linéaires semi-ordonnés pour lesquels on a (2): I) $y>0$ exclue $y=0$; II) pour tous $y_1>0$, $y_2>0$, on a $y_1+y_2>0$; III) pour chaque y il existe un $\bar{y}\geqslant 0$ tel que $\bar{y}-y\geqslant 0$; IV) pour tout élément $y>0$ et tout nombre $\lambda>0$, on a $\lambda y>0$. On appelle ces espaces K_4^-. Si au lieu de III on a III$_a$) pour chaque y il existe un $\bar{y}=\sup(y, 0)$, cet espace est nommé K_4.

On peut (2) introduire dans les espaces considérés les notions $<$ et inf dont nous notons les propriétés évidentes: $-y_1<-y_2$ est équivalent à $y_1>y_2$; $\inf(\{y_i\})=-\sup(\{-y_i\})$.

Nous dirons que deux espaces linéaires semi-ordonnés Y_1 et Y_2 sont isomorphes, s'il existe une transformation biunivoque qui conserve l'ordre et les relations linéaires.

Nous appellerons espace Euclidien n-dimensionnel semi-ordonné (R_n) l'espace Euclidien dans lequel $x(x_1, x_2, \ldots x_n)$ est positif, si $x_i\geqslant 0$ $(i=1, 2, \ldots n)$ et $x_i>0$ au moins pour un i. Il est évident que l'étude des espaces linéaires ayant un nombre fini de dimensions est équivalente à celle des espaces Euclidiens semi-ordonnés d'une manière quelconque. Il est facile à démontrer que pour qu'un espace linéaire semi-ordonné soit K_4^-, il faut et il suffit que pour $M=\mathcal{E}(x>0)$ nous ayons: a) M est un cône avec un sommet en 0 et $0\overline{\in}M$; b) M est convexe; c) pour chaque droite p il existe une droite parallèle p' $(p'\,\|\,p)$ telle que $p'M$ contient au moins deux éléments divers. Si Y est un espace n-dimensionnel, on peut remplacer c) par c'): c') il n'existe pas d'ensemble linéaire $(n-1)$-dimensionnel qui contienne M; ceci peut être écrit en termes topologiques

$M \neq 0$ (tous nos termes topologiques se rapportent à la topologie déterminée par les coordonnées dans l'espace ayant un nombre fini de dimensions).

Nous appellerons cône déterminant positif l'ensemble des éléments positifs et l'élément 0 et nous le désignerons par $P_0^+(Y)$. Nous donnons le même nom de cône déterminant aux ensembles

$$P_0^- = \underset{y}{\mathcal{E}}(-y \in P_0^+) = \underset{y}{\mathcal{E}}(y \leqslant 0); \quad P_y^{\pm} = P_0^+ + y.$$

Nous étudierons les cônes déterminants non seulement comme des ensembles de points, mais aussi comme ceux des rayons partant du sommet. En vertu de a) et b) deux rayons de cette nature dirigés dans les sens opposés ne peuvent pas appartenir à un même cône.

Notons que pour l'existence de $c = \sup(a, b)$ il faut et il suffit que

$$P_c^+ = P_a^+ \cdot P_b^+.$$

Par conséquent III_a est équivalent à la condition que la partie commune de deux cônes déterminants positifs quelconques (négatifs quelconques) soit un cône déterminant. S'il y a dans l'espace n-dimensionnel linéaire semi-ordonné Y_n un élément négatif vers lequel converge (par coordonnés) une suite d'éléments positifs, nous disons que Y_n est un espace **singulier**.

Deux exemples d'espaces singuliers K_4: 1) dans un plan le point est considéré comme un élément positif, si $x > 0$ ou $x = 0$, $y > 0$; 2) dans l'espace à trois dimensions le cône déterminant positif est l'ensemble des éléments pour lesquels $y > 0$, $z \geqslant 0$ ou $y = 0$, $x \geqslant 0$, $z \geqslant 0$.

Lemme. *Si Y_n est un espace n-dimensionnel K_4 et non-singulier, $P_0^+(Y_n)$ est fermé.*

Nous omettons la démonstration.

Soit M un sous-ensemble d'un espace k-dimensionnel, désignons par $f(M)$ la frontière de M. Nous dirons qu'une surface (surface de cône) est **linéairement décomposée** par deux points a, b (deux rayons $\vec{a}, \vec{b}$), si pour chaque point c (chaque rayon $\vec{c}$) de la surface, ou bien $[a, b]$ ($[\vec{a}, \vec{b}]$) ou bien $[b, c]$ ($[\vec{b}, \vec{c}]$) appartiennent à la surface.

Lemme fondamental. *Y_n étant un espace n-dimensionnel K_4^- et $a > 0$, $b > 0$, inf $(a, b) = 0$, on a: $f(P_0^+)$ est linéairement décomposé par les rayons $(0 \to a)$, $(0 \to b)$.*

Nous avons d'après $0 = \inf(a, b)$ que $P_0^- = P_a^- \cdot P_b^-$ et on voit aisément que $f(P_0^-) \subset f(P_a^-) + f(P_b^-)$.

Prenons un élément $y \in f(P_0^+) - (0)$; alors $-y \in f(P_0^-)$ et

$$-y \in f(P_a^-) \qquad (1_1)$$

ou bien

$$-y \in f(P_b^-). \qquad (1_2)$$

Si l'on a (1_1), on voit que $[0, -\lambda y]$ $\lambda > 1$ est contenu dans l'ensemble convexe $\overline{P_a^-}$, le point intérieur de ce segment appartient à la frontière $\overline{P_a^-}$ (la frontière de l'ensemble convexe étant identique à celle de leur fermeture). Alors tout le segment appartient à la frontière, et

$(0 \to -y) \subset f(P_a^-)$ par analogie $(a \to -\lambda y) \subset f(P_a^-)$ pour tout $\lambda > 0$; en d'autres termes l'ensemble de ces rayons $[(a \to 0), (a \to a-y)] \subset f(P_a^-)$ d'où

$$[(0 \to a), \quad (0 \to y)] \subset f(P_0^+). \tag{2_1}$$

Si (1_2) a lieu, nous obtenons d'une manière analogue

$$[(0 \to b), \quad (0 \to y)] \subset f(P_0^+) \tag{2_2}$$

c. q. f. d.

De la notion de supremum on peut tirer: 1) S'il existe $\inf M$ pour l'ensemble M, $M \subset P_y^+$ et s'il est impossible de transporter parallèlement P_y^+ dans une direction qui lui appartient de telle manière que l'ensemble M ne sorte pas du cône déterminant, on a $y = \inf M$. 2) Si $y = \inf M$, on a $M \subset P_y^+$ et on ne peut transporter le cône déterminant en conservant cette relation que dans des directions appartenant au cône déterminant négatif. Pour un espace ayant un nombre fini de dimensions on a: 3) Pour $x \in f(P_y^+)$ on ne peut transporter $\overline{P}_y^+$ en y conservant x que dans les directions appartenant au hyperplan d'appui passant par x.

Théorème I. *Y_n étant un espace K_4 non singulier n-dimensionnel, Y_n est isomorphe à R_n.*

A l'aide du lemme on peut démontrer qu'il existe un hyperplan L $(n-1)$-dimensionnel pour lequel $V = P_0^+ \cdot L$ est un corps convexe, borné, $(n-1)$-dimensionnel. Soit a un point, $a \in f(V)$ (la frontière relative à L) et supposons qu'il ne passe par a qu'un seul hyperplan d'appui T (un hyperplan tangent). Il existe un autre hyperplan T' d'appui pour V, $T' \| T$. Si $b \in T' \cdot V$, nous avons en vertu de 1 et 3: $\inf(a, b) = c$. Revenant au lemme fondamental, on voit que pour $-y = -a$ (1_2) n'est pas juste parce qu'il entraînerait (2_2) et ensuite $T' = T$. D'où $-a \in \overline{P_b^-}$ et il existe donc $U(-a)$, une sphère de centre $-a$, $U(-a) \subset \overline{P_b^-}$. Désignant par $U(a) = \underset{x}{\mathcal{G}}(-x \in U(-a))$, nous voyons aisément que pour chaque $x \in f(V)\, U(a)$ on a $[a, x] \subset f(V)$, donc on obtient $f(V)\, U(a) \subset T$.

Remarquons que pour chaque corps convexe, borné, $(n-1)$-dimensionnel il existe n hyperplans tangents indépendants $T_1, T_2, \ldots, T_n$. On sait de la théorie des corps convexes que l'ensemble des points de la frontière du corps convexe, où il existe un hyperplan tangent, est partout dense sur cette frontière (3). Cela étant, il est facile à construire les hyperplans tangents en question. Le premier T_1 est pris arbitrairement. Si l'on a construit k hyperplans tangents $T_1 \ldots T_k$, $k < n$, prenons un point $x \in T_1\, T_2 \ldots T_k$ (l'espace est considéré comme projectif) et un point y intérieur du corps. La droite $L(x, y)$ a deux points communs avec la frontière du corps, soit z l'un d'eux $(z \neq x)$. Le hyperplan tangent T_{k+1} passant par le point de la frontière assez proche à ce point ne peut pas passer par x. Il est manifeste que les hyperplans construits de telle manière sont indépendants, autrement dit $T_1 T_2 \ldots T_n$ est vide. Soit x_i $(i = 1, 2, \ldots n)$ le point contact du hyperplan T_i. Il existe donc un $U(x_i)$ pour lequel $f(V)\, U(x_i) \subset T_i$. Projetons L de 0 et désignons par $\mathbf{T}_i$ le hyperplan $(n-1)$-dimensionnel correspondant à T_i. On a $f(P_0^+)\, U(x_i) \subset \mathbf{T}_i$, donc x est un point intérieur de $P_0^+ \mathbf{T}_i$ et nous obtenons en vertu des propriétés 3), 1), 2)

$$P_0^+ \cdot \prod_{i \neq j} \mathbf{T}_i = \vec{x_j},$$

ou $\vec{x_j}$ est un rayon qui sort de 0. Cela nous donne

$$P_0^+ = [\vec{x_1}, \vec{x_2}, \ldots \vec{x_n}]$$

ce qui signifie que Y_n est isomorphe à R_n.

L'espace linéaire semi-ordonné Y est nommé non-singulier si chacun de ses sous-espaces linéaires ayant un nombre fini de dimensions et contenant 0 est non-singulier. On peut d'une manière analogue à celle du lemme démontrer que si Y est un espace K_4 non-singulier et L son sous-espace n-dimensionnel contenant 0, alors $P_0^+(L) = L \cdot P_0^+(Y)$ est fermé.

Théorème II. *Y est un espace non-singulier K_4 et*

$$F_\alpha = F_\alpha(x_1, \ldots x_k) \quad (\alpha = 1, 2, \ldots t)$$

sont des expressions qui se composent d'éléments et de signes $+$, $-$, inf, sup *en nombre fini; si les relations*

$$F_\alpha = 0 \qquad (\alpha = 1, 2, \ldots t-1) \tag{3}$$

entraînent

$$F_t = 0 \tag{4}$$

quand les x_i sont des nombres réels, la même assertion a lieu quand x_i sont des éléments de Y.

Le théorème est évident quand Y est isomorphe à R_n. Grâce aux propriétés d'extremum: $\sup(\{y_i\}) = -\inf(\{-y_i\})$ et $\inf(a, b, c) = \inf[a, \inf(b, c)]$, on peut reduire F_α à $\bar{F}_\alpha = F_\alpha$ dans lesquels sup manque et on ne prend inf que pour deux éléments. Prenons dans Y des éléments arbitraires $x_1, x_2, \ldots x_k$ et ajoutons tous les éléments $x_{k+1}, \ldots x_l$ qui présentent les expressions composées des éléments $x_1, \ldots x_k$ et qui entrent dans $\bar{F}_\alpha (\alpha = 1, 2, \ldots, t)$. Introduisons l'espace semi-ordonné Y_m, $Y_m = L(x_1, \ldots x_l, 0)$ dans lequel l'ordre est transféré de Y. Tous les groupes de trois nombres pris de $x_1, \ldots x_k, x_{k+1}, \ldots x_l$ et qui sont liés par l'égalité $\inf(x_\beta, x_\gamma) = x_\delta$ seront désignés par a_i^1, a_i^2, b_i $(i = 1, \ldots, p)$; alors

$$\inf(a_i^1, a_i^2) = b_i. \tag{5}$$

Posons $a_i^\varepsilon - b_i = c_i^\varepsilon$. Alors (5) sera équivalent à

$$\inf(c_i^1, c_i^2) = 0. \tag{6}$$

Construisons, de même que dans la construction de Y_m, un espace semi-ordonné $L = L(c_1^1, c_2^1, \ldots, c_p^1, c_1^2, \ldots, c_p^2, 0)$. Il est manifeste que l'égalité (6) est vraie dans Y_m et dans L.

Partageons i en deux classes: dans la première nous portons les i pour lesquels $c_i^\varepsilon = 0$ ($\varepsilon = 1$ ou 2), dans la deuxième tous les autres. En vertu de la méthode de sa construction, L est un espace K_4^- non-singulier. $P_0^+(L)$ est fermé. En vertu du lemme fondamental nous voyons que chaque hyperplan T tangent à $P_0^+(L)$ passe par c_i^ε, où $\varepsilon = 1$ ou bien 2 et i appartient à la deuxième classe. Construisons comme dans le théorème précédent n hyperplans tangents indépendents (où n est le nombre de dimensions de L). Ces hyperplans tangents partagent L en 2^n cônes. Désignons par P celui d'eux qui contient $P_0^+(L)$ et prenons-le pour le

cône déterminant d'un espace semi-ordonné nouveau L. $\overline{L}$ est isomorphe à R_n. En vertu de 3) et 1) nous voyons que (6) est vrai dans $\overline{L}$.

Si m est le nombre de dimensions de Y_m, nous prenons dans Y_m $m-n$ rayons $\vec{x}_j\,(j=1,\ldots m-n)$ partant de 0 et indépendents avec L. $\overline{Y}_m$ étant un espace semi-ordonné que l'on obtient de Y_m si l'on prend $[\vec{x}_1, \ldots \vec{x}_{m-n}, P]$ pour cône déterminant, $\overline{Y}_m$ est isomorphe à R_m. Dans $\overline{Y}_m$ (6) est vrai et par conséquent (5) l'est également.

Soient maintenant $x_1, x_2, \ldots x_k$ des éléments de Y pour lesquels (3) a lieu dans Y. Cette équation est satisfaite pour ces éléments également dans $\overline{Y}_m$, parce que $\overline{F}_\alpha(x_1, x_2, \ldots, x_k)$ est identique dans Y et dans $\overline{Y}_m$. D'où (4) est satisfait dans $\overline{Y}_m$ (parce que $\overline{Y}_m$ est isomorphe à R_m) et (4) est vrai dans Y.

Remarque. Si dans (3) et (4) on remplace les signes $=$ par $\geqslant$, le théorème reste juste. En effet, $F_\alpha \geqslant 0$ est équivalent à $\inf(F_\alpha, 0)=0$. Remarquons que l'espace linéaire Y ne peut pas être singulier, si l'on peut définir dans Y une convergence pour laquelle:

A) pour chaque élement x $\lambda_n x \to 0$, si $\lambda_n \to 0$;

B) si $x_n \to x$ et $y_n \to y$, on a $x_n + y_n \to x + y$;

C) si $x_n \to x$ et $x_n > 0$, on a $x \not< 0$.

Ces conditions ont lieu pour la convergence dans les espaces B_1 [4] et pour les espaces semi-ordonnés K_5 [2]. L'espace semi-ordonné s'appelle K_5 si, sauf les axiomes I — IV, on a V) $y_1, y_2, \ldots y_n, \ldots$ étant les éléments pour lesquels on a $y_i \leqslant \overline{y}$ $(i=1, 2\ldots)$ il existe alors

$$\overline{y} = \sup(y_1, y_2, \ldots y_n, \ldots).$$

Institut de Mathématiques et de Mécanique.
Université de Leningrad.

Manuscrit reçu
le 26. III. 1939.

LITTÉRATURE CITÉE

[1] Hausdorff, Grundzüge der Mengenlehre (1914). [2] L. V. Kantorovitch, Rec. Math., 2 (44), 1 (1937). [3] Reidemeister, Math. Ann., 88 (1921). [4] L. Kantorovitch, C. R. Acad. Sci. URSS, III (XII) (1936).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

MATHÉMATIQUES

APPROXIMATION DES POLYÈDRES SIMPLEMENT CONNEXES AU MOYEN DE SURFACES RATIONNELLES

Par M. FRANK

(Présenté par S. N. Bernstein, de l'Académie, le 25. III. 1939)

Plaçons un polyèdre simplement connexe de telle manière que l'un de ses sommets S_0 se trouve à l'origine des coordonnées. Nous supposons que toutes ses faces sont des triangles; dans ce but, si l'une des faces a plus de trois côtés, nous menons des diagonales qui la décomposent en triangles. Soit λ_0 le nombre des faces qui se rencontrent en S_0. Excluons toutes ces faces et faisons une transformation biunivoque et réciproque de la surface restante du polyèdre, qui présente un domaine ouvert simplement connexe, sur le plan (u, v).

On obtient sur le plan (u, v) un domaine ω formé d'un réseau de triangles et borné par un polygone à λ_0 côtés. Les sommets des triangles peuvent toujours, comme il est facile à voir, être disposés de manière que leurs coordonnées ne prennent que des valeurs entières.

Numérotons par les mêmes numéros tous les $f-\lambda_0$ faces du polyèdre et tous les triangles qui leur correspondent. Il correspond à une face M_k ayant pour sommets M_{k1}, M_{k2}, M_{k3} un triangle $N_k(N_{k1}, N_{k2}, N_{k3})$ et on a d'ailleurs

$$\left.\begin{array}{l}\text{pour } M_{ks}\colon\ \xi=a_{ks},\ \eta=b_{ks},\ \zeta=c_{ks}\\ \text{pour } N_{ks}\colon\ u=\alpha_{ks},\ v=\beta_{ks}\end{array}\right\}\ s=1,2,3.$$

L'équation du plan de la face M_k peut alors être écrite dans la forme

$$\xi_k=\frac{1}{D_k}\begin{vmatrix} u & v & 1 & 0\\ \alpha_{k1} & \beta_{k1} & 1 & a_{k1}\\ \alpha_{k2} & \beta_{k2} & 1 & a_{k2}\\ \alpha_{k3} & \beta_{k3} & 1 & a_{k2}\end{vmatrix}\quad \text{où } D_k=\begin{vmatrix}\alpha_{k1} & \beta_{k1} & 1\\ \alpha_{k2} & \beta_{k2} & 1\\ \alpha_{k3} & \beta_{k3} & 1\end{vmatrix} \tag{1}$$

et on a des formules analogues pour η_k et ζ_k en remplaçant les lettres a par b et c. On numérote les sommets de chaque face de manière à avoir $D_k>0$.

D'autre part, en considérant le polyèdre donné comme un polycoryphe à e sommets, numérotons tous ses sommets et les sommets du réseau obtenu par la transformation. Le nombre des sommets intérieurs du réseau sera évidemment égal à $e-\lambda_0-1$.

Le sommet S_i qui est le point de rencontre de λ_i faces a pour coordon-

nées a_i, b_i, c_i, et le sommet correspondant de la transformation T_i a pour coordonnées α_i, β_i (il est évident que de cette manière un même sommet sera désigné de deux façons différentes, suivant que l'on le considère comme sommet d'une face du polyèdre ou comme sommet du polycoryphe).

Introduisons les notations:

$$\sigma_{k1}=\left[1-\frac{D_{k1}}{D_k}\right]^{D_k};\ \sigma_{k2}=\left[1-\frac{D_{k2}}{D_k}\right]^{D_k};\ \sigma_{k3}=\left[1-\frac{D_{k3}}{D_k}\right]^{D_k} \tag{2}$$

où D_k a la valeur (1) et

$$D_{k1}=\begin{vmatrix} u & \varphi & 1 \\ \alpha_{k2} & \beta_{k2} & 1 \\ \alpha_{k3} & \beta_{k3} & 1 \end{vmatrix};\ D_{k2}=\begin{vmatrix} \alpha_{k1} & \beta_{k1} & 1 \\ u & \varphi & 1 \\ \alpha_{k3} & \beta_{k3} & 1 \end{vmatrix};\ D_{k3}=\begin{vmatrix} \alpha_{k1} & \beta_{k1} & 1 \\ \alpha_{k2} & \beta_{k2} & 1 \\ u & \varphi & 1 \end{vmatrix} \tag{3}$$

c'est-à-dire les D_{ki} présentent le double des aires des triangles, dans chacun desquels l'un des sommets N_{k1}, N_{k2}, N_{k3} est remplacé par un point N à coordonnées variables u, φ; posons d'ailleurs

$$\tau_{p1}^{(i)}=\left[\frac{1-\dfrac{D_{p1}^{(i)}}{D_p^{(i)}}}{1+\dfrac{D_{p1}^{(i)}}{D_p^{(i)}}}\right]^{D_p^{(i)}};\quad \tau_{p2}^{(i)}=\left[\frac{1-\dfrac{D_{p2}^{(i)}}{D_p^{(i)}}}{1+\dfrac{D_{p2}^{(i)}}{D_p^{(i)}}}\right]^{D_p^{(i)}} \tag{4}$$

où $D_p^{(i)}$ est le double de l'aire de l'un des triangles adjacents à la surface T_i, et $D_{p1}^{(i)}$ et $D_{p2}^{(i)}$ sont respectivement le double des aires des triangles dont l'un des sommets, ne coïncidant pas avec le sommet T_i, est remplacé par le point $N(u, \varphi)$. On numérote les sommets de chaque triangle $N_p^{(i)}$ de manière que l'on ait $D_p^{(i)}>0$.

Prenons les équations

$$x=\sum_{k=1}^{k=f-\lambda_0}\frac{\xi_k}{1+(\sigma_{k1})^{4n}+(\sigma_{k2})^{4n}+(\sigma_{k3})^{4n}}+ \\ +\sum_{i=1}^{i=e-\lambda_0-1}a_i\left[1-\sum_{p=1}^{p=\lambda_i}\frac{1}{1+(\tau_{p1}^{(i)})^{2n}+(\tau_{p2}^{(i)})^{2n}}\right] \tag{5}$$

et des équations analogues pour y et z, que l'on obtient en remplaçant ξ_k par η_k et ζ_k, a_i par b_i et c_i.

On peut démontrer que les équations (5) représentent une surface qui s'approche uniformément du polyèdre et d'ailleurs dans tout le domaine Ω l'ordre de l'approximation est supérieur ou égal à $\left(\frac{1}{n}\right)$.

Désignons la première somme (5) par

$$x^{(1)}=\sum_{k=1}^{k=f-\lambda_0}x_k^{(1)}$$

et la seconde par

$$x^{(2)}=\sum_{i=1}^{i=e-\lambda_0-1}a_i\left[1-\sum_{p=1}^{p=\lambda_i}\mu_p\right].$$

1. Si le point N se trouve à l'intérieur du triangle N_k, on a

$$0<\frac{D_{ks}}{D_k}<1;\ (\sigma_{ks})^{4n}<e^{-4nD_{ks}};\ s=1,2,3,$$

et ainsi $x_k^{(1)}=\xi_k+O(e^{-n\delta})$, où δ est une quantité qui ne surpasse aucune des valeurs $4D_{ks}$.

Pour les autres membres de la somme $x^{(1)}$ correspondant aux autres triangles, pour lesquels le point N se trouve hors de leur aire, on peut facilement démontrer que l'un au moins des nombres D_{l1}, D_{l2}, D_{l3}, présentant le double des aires, doit être négatif et d'ailleurs en module supérieur ou égal à une certaine quantité finie positive $\frac{\delta}{4}$.

Il en résulte que l'une des trois valeurs

$$(\sigma_{ls})^{4n} > 2^{n\delta},$$

donc

$$x^{(1)} = \xi_k + O(2^{-n\delta}).$$

2. Si le point N se trouve sur un côté du triangle N_k appartenant aussi au triangle N_{k+1}, alors dans les expressions de $x_k^{(1)}$ et $x_{k+1}^{(1)}$ l'une des valeurs σ devient égale à 1 et

$$x_k^{(1)} + x_{k+1}^{(1)} = \frac{\xi_k}{2 + O(e^{-n\delta})} + \frac{\xi_{k+1}}{2 + O(e^{-n\delta})} = \xi + O(e^{-n\delta}),$$

car dans ce cas on a $\xi_k = \xi_{k+1} = \xi$.

Les autres membres de la somme $x^{(1)}$ donneront dans ce cas, comme dans le précédent, une quantité d'ordre $(2^{-n\delta})$.

La pire approximation est obtenue, comme il est facile à voir, quand le point N se trouve près d'un côté du triangle et d'ailleurs sa distance elle-même est d'ordre $\left(\frac{1}{n}\right)$. Dans ce cas l'un des D_{ks}, par exemple, $D_{k1} = \frac{\alpha}{n}$, où $\alpha > 0$, et pour le triangle voisin l'un des D, par exemple, $D_{k+1,2} = -\frac{\alpha}{n}$, d'où

$$(\sigma_{k,1})^{4n} \approx e^{-4\alpha}\left[1 - \frac{2\alpha^2}{nD_k}\right]$$

et

$$(\sigma_{k+1,2})^{4n} \approx e^{4\alpha}\left[1 - \frac{2\alpha^2}{nD_{k+1}}\right],$$

donc

$$x_k^{(1)} + x_{k+1}^{(1)} = \frac{\xi_k}{1 + e^{-4\alpha} + O\left(\frac{1}{n}\right)} + \frac{\xi_{k+1}}{1 + e^{4\alpha} + O\left(\frac{1}{n}\right)},$$

et comme dans ce cas $\xi_{k+1} = \xi_k + O\left(\frac{1}{n}\right)$, on a $x_k^{(1)} + x_k^{(2)} = \xi_k + O\left(\frac{1}{n}\right)$, et tous les autres membres seront des quantités d'ordres supérieurs.

Pour déterminer la seconde somme $x^{(2)}$, si le point N ne coïncide avec aucun des sommets, il faut tenir compte de ce que le point N se trouve par rapport à chaque sommet ou bien à l'intérieur de l'un des angles, formé par deux demi-droites sortant de ce sommet et dont les directions coïncident avec celles des deux côtés d'un triangle, ou bien sur l'une de ces droites.

Dans le premier cas pour l'un des triangles, par exemple $N_p^{(i)}$, les deux aires $D_{p1}^{(i)} > 0$ et $D_{p2}^{(i)} > 0$, donc, comme il est facile à voir $(\tau_{p1}^{(i)})^{2n} < 2^{-n\delta}$ et $(\tau_{p2}^{(i)})^{2n} < 2^{-n\delta}$, et pour tous les autres triangles l'une des valeurs $D_{qs}^{(i)} < 0$ et $(\tau_{qs}^{(i)})^{2n} > 2^{n\delta}$, $(s = 1, 2)$, et ainsi $\mu_p^{(i)} = 1 + O(2^{-n\delta})$ et pour les autres $\lambda^i - 1$ membres de la somme $\sum_{q=1}^{q=\lambda_i} \mu_q^{(i)}$, où $q \neq p$, on

a par exemple $\mu_q^{(i)} = O(2^{-n\delta})$; et comme cela a lieu pour tous les sommets, on a $x^{(2)} = O(2^{-n\delta})$.

On a le même résultat si N se trouve sur un côté ou bien sur son prolongement, mais dans ce cas la somme de deux μ pour deux triangles voisins donne

$$\mu_p + \mu_{p+1} = 1 + O(2^{-n\delta})$$

et tout le reste n'est pas modifié.

Enfin, si le point N s'approche du côté et sa distance est d'ordre $\left(\frac{1}{n}\right)$, il est facile à démontrer que

$$x^{(2)} = O\left(\frac{1}{n}\right).$$

3. Le dernier cas est celui où le point N se trouve au sommet, c'est-à-dire $\xi = a_i$; alors pour tous les triangles entourant ce sommet dans l'expression de $x^{(1)}$ deux des trois valeurs σ_{ks} $(s = 1, 2, 3)$ deviennent égales à 1, et la troisième s'annulle, donc

$$x^{(1)} = \lambda_i \frac{a_i}{3} + O(2^{-n\delta}).$$

Mais en même temps pour les mêmes triangles tous les τ deviennent égaux à 1 et

$$x^{(2)} = a_i \left(1 - \lambda_i \frac{1}{3}\right) + O(2^{-n\delta})$$

d'où $x = x^{(1)} + x^{(2)} = a_i + O(2^{-n\delta})$.

D'une manière analogue, dans le cas où le point N se trouve à une distance d'ordre $\left(\frac{1}{n}\right)$ du sommet T_i, ce qui est le pire cas, nous obtenons

$$x^{(1)} + x^{(2)} = a_i + O\left(\frac{1}{n}\right).$$

L'approximation vers les arêtes qui forment la frontière du domaine Ω est encore plus faible, à savoir d'ordre $\frac{1}{n^\beta}$, où β est un nombre inférieur à 1, mais aussi proche de 1 que l'on voudra.

Le long des faces et des arêtes ayant pour sommet commun S_0, ainsi qu'au sommet S_0 lui-même, l'ordre d'approximation est supérieur ou égal à $\left(\frac{1}{n}\right)$.

Pour une surface polyédrale simplement connexe non-fermée, en faisant deux fois le tour d'une telle surface, on obtient une approximation au moyen des mêmes formules. Pour un polyèdre multiplement connexe il faut faire des coupures pour le transformer en une surface simplement connexe non-fermée et obtenir ensuite une approximation.

La même méthode permet d'obtenir l'approximation d'une fonction $z = f(x, y)$ définie dans un domaine $a < x < b$, $c < y < d$, ne prenant qu'une seule valeur pour chaque couple de valeurs des variables et possédant les dérivées premières continues et les dérivées du second ordre bornées.

Un exposé détaillé contenant toutes les démonstrations sera publié dans les Travaux de l'Institut Industriel à Leningrad.

Manuscrit reçu
le 26. III. 1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

MATHEMATICS

ON THE ORTHOGONALITY OF A SYSTEM OF POLYNOMIALS ON SEVERAL CONTOURS

By J. GERONIMUS

(Communicated by S. N. Bernstein, Member of the Academy, 25. III. 1939)

Consider an analytic contour C in the z-plane and a weight function $n(z)$ positive and continuous on C; then there exists an analytic function $D(z)$ regular and non-vanishing outside C ($z=\infty$ inclusive) with the boundary property (1):

$$\lim_{z\to z_0} |D(z)|^2 = n(z_0), \qquad (z - \text{outside } C, \quad z_0 - \text{on } C). \tag{1}$$

Let the function $w=\gamma(z)$ map the region outside C conformally and one-to-one on the region $|w|>r_0$ so that the points at infinity correspond to each other and

$$\lim_{z\to\infty} \frac{\gamma(z)}{z} = 1. \tag{2}$$

Denote by $C_r\,(r>r_0)$ the image in the z-plane of the circle $|w|=r>r_0$ (level curve).

Theorem I. *The ratios of the moments*

$$\frac{\gamma_k(r)}{\gamma_0(r)}, \quad (k=1, 2, \ldots), \tag{3}$$

where

$$\gamma_k(r) = \int_{C_r} |D(z)|^2 z^k \, |dz|, \quad (k=0, 1, 2, \ldots), \quad (r \geqslant r_0) \tag{4}$$

are independent of r if and only if the contour C and the weight function $n(z)$ are of the following types *:

I. *The contour C is a curve of the family C_r,*

$$C_r: \qquad \left| \frac{G_k(z) + \sqrt{G_k^2(z) - 4\alpha^{2k}}}{2} \right| = r^k, \tag{5}$$

[$G_k(z)$ being an arbitrary polynomial of degree k with the coefficient of z^k equal to 1]; the contour C corresponds to a value $r=r_0$, for which the

* The weight function $n(z)$ differs from $|D(z)|^2$ by factors which are constant on C_r.

smallest convex polygon containing on or within it all roots of $G_k(z)$ *lies inside* C_{r_0}, *and* $r_0 > |\alpha| \geqslant 0$; *the weight function* $n(z)$ *is given by the formulae*

$$n(z) = \left| \sqrt{\frac{G_k(z) + 2\alpha^k}{G_k(z) - 2\alpha^k}}\, G_k'(z) \right|, \quad 0 < |\alpha| < r_0; \tag{6'}$$

$$n(z) = \left| \frac{G_k'(z)}{[G_k(z) - \lambda^k]^2} \right|, \quad \alpha = 0, \quad |\lambda| \leqslant r_0. \tag{6''}$$

II. *The contour* C *is arbitrary and* $n(z) = |\gamma'(z)|$.

Consider now two kinds of orthogonality of a system of polynomials $\{P_n(z; r)\}$ on the contours $C_r (r \geqslant r_0)$:

$$\int_{C_r} |D(z)|^2 P_n(z; r) \overline{P_m(z; r)}\, |dz| \quad \begin{cases} = 0, & n \neq m; \\ \neq 0, & n = m; \end{cases} \tag{A}$$

$$\int_{C_r} |D(z)|^2 P_n(z; r) P_m(z; r)\, |dz| \quad \begin{cases} = 0, & n \neq m; \\ \neq 0, & n = m^*. \end{cases} \tag{B}$$

Consider also the functions of the second kind

$$Q_n(y; r) = \int_{C_r} \frac{|D(z)|^2 \overline{P_n(z; r)}\, |dz|}{y - z}, \quad (n = 0, 1, \ldots), \quad (y - \text{outside } C_r), \tag{7}$$

the system $\{P_n(z; r)\}$ being A-orthogonal.

Theorem I is closely connected with the question about the simultaneous orthogonality of a system $\{P_n(z; r_0)\}$ on all $C_r (r > r_0)$. In fact, we have the following

Theorem II. *A system of polynomials which is B-orthogonal on* C *will be B-orthogonal on all* $C_r (r > r_0)$, *if and only if the contour* C *is given by* (5) *and the weight-function* $n(z)$ *by* (6'), *or if the contour* C *is arbitrary and* $n(z) = |\gamma'(z)|$.

Corollary I. In the case when $Q_0(z; r_0)$ is a rational function, it is impossible to construct a system of polynomials B-orthogonal on C; therefore we exclude the case when $n(z)$ is given by (6'') and the case when $\frac{\gamma'(z)}{\gamma(z)}$ is a rational function.

Corollary II. The independence of r of the ratios

$$\frac{\gamma_k(r)}{\gamma_0(r)}, \quad (k = 1, 2, \ldots), \quad (r > r_0),$$

is necessary, but not sufficient for the A-orthogonality on all $C_r (r > r_0)$ of a system of polynomials which is A-orthogonal on C; this condition is equivalent to the following one for the functions of the second kind **:

$$Q_0(y; r) = f_0(r)\, Q_0(y; r_0), \quad (r > r_0). \tag{8}$$

Both necessary and sufficient conditions are given by

Theorem III. *The conditions*

$$Q_0(y; r) = f_0(r)\, Q_0(y; r_0); \quad Q_1(y; r) = f_1(r)\, Q_1(y; r_0), \quad (r > r_0), \tag{9}$$

---

* The B-orthogonality has been considered by J. Walsh and G. Merriman [3] only recently.

** $f_n(r)$ and $\varphi_n(r)$ in (11) $(n = 0, 1, 2 \ldots)$ are functions of r.

are both necessary and sufficient for the A-orthogonality on all C_r of a system of polynomials which is A-orthogonal on C.

The problem of the A-orthogonality of a system of polynomials on all $C_r\,(r > r_0)$ has been solved by G. Szegö [1] who found five distinct types of polynomials possessing this property. Theorem III gives the solution of the same problem under less restricting assumptions.

Indeed, we see from the expansion *

$$\frac{1}{y-z} = \sum_{n=0}^{\infty} \frac{P_n(z;r)\,Q_n(y;r)}{h_n(r)}, \quad h_n(r) = \int_{C_r} |D(z)|^2\,|P_n(z;r)|^2\,|dz|, \tag{10}$$

that the conditions of A-orthogonality on all $C_r\,(r > r_0)$ of a system $\{P_n(z;r_0)\}$ which is A-orthogonal on C, namely

$$P_n(z;r) = \varphi_n(r)\,P_n(z;r_0), \quad (n=0,1,2,\ldots), \quad (r > r_0), \tag{11}$$

yield at once the conditions for the functions of the second kind:

$$Q_n(y;r) = f_n(r)\,Q_n(y;r_0), \quad (n=0,1,2,\ldots), \quad (r > r_0). \tag{12}$$

Our theorem III shows that the validity of these conditions for $n=0$ and $n=1$ implies their validity for all positive integral n.

Institute of Mathematics and Mechanics.
Kharkov State University.

Received
26. III. 1939.

REFERENCES

[1] G. Szegö, Trans. Amer. Math. Soc., **37** (1935). [2] J. Walsh a. G. Merriman, Duke Math. Journal, **3** (1937).

* This expansion converges uniformly, if y is outside or on $C_\rho\,(\rho > r > r_0)$, and is within or on C_r.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

MATHEMATICS

ON NORMED RINGS

By I. GELFAND

(Communicated by A. N. Kolmogoroff, Member of the Academy, 27. III. 1939)

Definition 1. The set R of elements $x, y, \ldots$ is called a normed ring, if: a) it is a complete normed space with multiplication by complex numbers; b) together with the elements x and y exists their product xy possessing the usual algebraical properties and continuous at least with respect to one of its factors; c) it contains a unit element e.

It may be shown that in R exists a norm equivalent to a given one and possessing the following properties:

$$\|xy\| \leqslant \|x\| \cdot \|y\|, \quad \|e\| = 1.$$

In what follows we consider only commutative rings.

Definition 2. A set of elements $x, y, \ldots \subset R$ is called an ideal I, if it possesses the following properties: a) if $x \in I$, $y \in I$, then $\alpha x + \beta y \in I$, where α and β are arbitrary complex numbers; b) if $x \in I$ and z is an arbitrary element of the ring R, then $xz \in I$.

The whole ring and the nul ideal, i. e. the ideal consisting of only one element 0, are trivial ideals. Every ideal different from these two we shall call non-trivial.

Definition 3. A non-trivial ideal not contained in any other non-trivial ideal we call a maximal ideal.

Theorem 1. *Every maximal ideal is closed (i. e. the set of elements contained in this ideal is closed in R).*

Theorem 2. *Every non-trivial ideal is contained in a maximal ideal.*

The ring of residues R/I, where I is a closed ideal, may be again made a normed ring. As is known, the elements of R/I are classes X, Y, ..., consisting of such elements x', x'', ... that $x' - x'' \in I$. The norm of the class X is defined in the following way: $\|X\| = \inf |x|$, where $x \in X$. It may be shown that in the ring of residues all axioms of a normed ring are satisfied.

Theorem 3. *The ring of residues to a maximal ideal is the corpus of complex numbers.*

Thus, to every element $x \in R$ and every maximal ideal M there corresponds a complex number $x(M)$, namely the number, which is correlated to x by the homomorphism $R \sim R/M$. Hence follows

Theorem 4. *To every element $x \in R$ there corresponds a function*

$x(M)$ defined on the set $\mathfrak{M}$ of all maximal ideals. To the sum of elements corresponds the sum of functions, to the product of elements — the product of functions.

The function $x(M)$ is bounded. In fact, $|x(M)| \leqslant \|x\|$. It may be shown that $\sup|x(M)| = \lim\limits_{n\to\infty} \sqrt[n]{\|x^n\|}$. Hence we obtain

Theorem 5. *If in the ring R there are no elements different from zero, for which $\sqrt[n]{\|x^n\|} \to 0$, then our ring is isomorphic to the ring of functions defined on the set of maximal ideals.*

We shall show that these functions $x(M)$ are continuous. To this end we must topologize the set $\mathfrak{M}$ of maximal ideals. We shall define the topology by a system of neighbourhoods.

Definition 4. A set K of elements of the ring R is called a set of generators of the ring R, if the smallest closed ring containing K is the whole ring R.

Definition 5. By the neighbourhood of the maximal ideal M_0 we shall mean the set of maximal ideals satisfying the inequalities

$$|x_i(M) - x_i(M_0)| < \varepsilon, \qquad (i = 1, 2, \ldots, n),$$

where ε and n are arbitrary and $x, \ldots, x_n$ are arbitrary elements of the set K of generators of the ring R.

The so obtained topology in $\mathfrak{M}$ does not depend on the choice of K. The set $\mathfrak{M}$ of maximal ideals in this topology proves to be a bicompact Hausdorff space and the functions $x(M)$ are continuous on it. As the result we obtain the following fundamental

Theorem 6. *Every abstract normed ring R may be homomorphically mapped in the ring of continuous functions defined on the Hausdorff bicompact space of maximal ideals of the ring R. For the isomorphism it is necessary and sufficient that the conditions of Theorem 5 should be satisfied.*

In some cases the topological properties of the set $\mathfrak{M}$ may be stated more precisely, namely:

1. If the ring R is separable, then the set $\mathfrak{M}$ is metrisable.

2. If the ring R is generated by a system of n generators, then the set $\mathfrak{M}$ is homeomorphic to a compact subspace of the n-dimensional complex space.

Theorem 7. *If in the ring R for each element x there is a complex-conjugated element, i. e. such an element y that $y(M) = \overline{x(M)}$, whatever be M from $\mathfrak{M}$, then each function continuous on the set $\mathfrak{M}$ is the limit of a uniformly convergent sequence of functions corresponding to the elements of the ring R*.*

Hence it follows that if in the ring R a norm is introduced in such a way that from the uniform convergence of the functions $x_n(M)$ follows the convergence of the norms of the elements x_n (for which it is, for instance, sufficient to demand that $\|x^2\| = \|x\|^2$), then the ring R consists of all functions continuous on $\mathfrak{M}$ (we assume that the conditions of theorem 7 are satisfied in the ring R).

Thus, in many cases the aggregate of functions $x(M)$ uniquely determines the ring. This follows from

---

* This theorem was proved by the author together with G. Šilov. The proof will be given in their common paper.

Theorem 8. *Let there be given two normed rings R_1 and R_2. Let the intersection of all maximal ideals in each of them be the nul ideal. Then from the algebraic isomorphism of the rings R_1 and R_2 follows their continuous isomorphism.*

The applications of the above results will be given elsewhere. A detailed exposition will be published in the «Recueil Mathématique de Moscou».

Institute of Mathematics.
Moscow State University.

Received
27. III. 1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

MATHÉMATIQUES

SUR LA MÉTRISATION DE CONVERGENCES DANS LES ESPACES LINÉAIRES

Par B. VULICH

(Présenté par S. N. Bernstein, de l'Académie, le 25. III. 1939)

Étant donné un espace linéaire topologique, le problème de la détermination d'une métrique qui conserve la topologie donnée a été étudié dernièrement par divers auteurs. A. Kolmogoroff (1) a établi les conditions, sous lesquelles on peut introduire une norme dans un espace linéaire topologique, et D. H. Hyers (2) et J. V. Wehausen (3) ont étudié l'introduction d'une métrique arbitraire invariante par rapport à l'opération d'addition. D'ailleurs, la convergence suivant la distance coïncide, bien entendu, avec la convergence «topologique». Or, il est évident, qu'on ne peut métriser de la façon habituelle qu'une convergence qui coïncide avec sa convergence (*) (la définition est donnée dans la suite). Mais, si une convergence (dans un ensemble linéaire) ne jouit pas de cette propriété, on peut poser la question si elle admet une métrisation plus générale (K-normation ou K-métrisation invariante). La présente Note est consacrée précisément à cette question. Un exposé détaillé et les démonstrations des résultats énoncés ci-dessous seront publiés ailleurs.

§ 1. Définition. Nous appellerons espace (L) linéaire un ensemble linéaire $X=\{x\}$, où l'on a défini la notion de convergence (γ) vers zero d'une suite $\{x_n\}$ de ses éléments $(x_n \xrightarrow{\gamma} 0)$, cette convergence vérifiant les conditions suivantes:

1) $x_n \xrightarrow{\gamma} 0$ et $y_n \xrightarrow{\gamma} 0$ impliquent $x_n + y_n \xrightarrow{\gamma} 0$;

2) $x_n \xrightarrow{\gamma} 0$ implique $\alpha x_n \xrightarrow{\gamma} 0$ (α est un nombre réel arbitraire);

3) $\alpha_n \longrightarrow 0$ (α_n sont des nombres réels) implique $\alpha_n x \xrightarrow{\gamma} 0$ pour tout $x \in X$;

4) $x_n \xrightarrow{\gamma} 0$ et $x_n = x$ pour tout n impliquent $x=0$;

5) si $x_n \xrightarrow{\gamma} 0$ et si $n_1 < n_2 < \ldots \longrightarrow \infty$, on a $x_{n_i} \xrightarrow{\gamma} 0$;

6) si $x_n \xrightarrow{\gamma} 0$ et $y_n \xrightarrow{\gamma} 0$, et si $z_n = x_m$ pour $n = 2m-1$ et $z_n = y_m$ pour $n=2m$, alors on a $z_n \xrightarrow{\gamma} 0$;

7) si $x_n \xrightarrow{\gamma} 0$, et si la suite $\{y_n\}$ est obtenue au moyen de quelques transpositions d'éléments d'une suite $\{x'_k\}$, où $x'_1 = \ldots = x'_{k_1} = x_1$, $x'_{k_1+1} = \ldots = x'_{k_2} = x_2, \ldots, x'_{k_{n-1}+1} = \ldots = x'_{k_n} = x_n, \ldots$ et où $k_1 < k_2 < \ldots$ sont des nombres entiers quelconques, on a $y_n \xrightarrow{\gamma} 0$.

Posons dans un espace (L) linéaire $x_n \xrightarrow{\gamma} x$, si $x_n - x \xrightarrow{\gamma} 0$. D'ailleurs. l'unicité de la limite est assurée, et l'on peut vérifier sans aucune difficulté que la limite (γ) jouit de propriétés semblables à 1) — 7). En outre, l'égalité $x_n = x$ pour tout n implique $x_n \xrightarrow{\gamma} x$ (c.-à-d. $x \xrightarrow{\gamma} x$). Ainsi, un espace (L) linéaire est un cas particulier d'un espace (L) de M. Fréchet [(4), p. 164, (5), p. 230]. ·

En utilisant la convergence donnée (γ), on peut introduire dans l'ensemble X une certaine topologie, à savoir, on peut considérer comme fermé un ensemble $M \subset X$, si les relations $x_n \xrightarrow{\gamma} x$ et $x_n \in M$ $(n = 1, 2, \ldots)$ impliquent $x \in M$. L'ensemble X devient alors un espace topologique au sens de MM. Riesz et Kuratowski (6). On peut définir dans cet espace la convergence «topologique», à savoir $x_n \xrightarrow{t} x$, si tout voisinage du point x contient presque tous les x_n. Mais cette convergence topologique est en général plus large que la convergence (γ) *.

§ 2. Définition. Supposons que deux convergences (γ) et (δ) soient définies dans un ensemble quelconque. Nous appellerons convergence (*) pour la convergence (γ) la convergence (δ), si la relation $x_n \xrightarrow{\delta} x$ équivaut au fait qu'on peut extraire de chaque suite partielle $\{x_{n_i}\}$ une autre suite partielle $x'_n \xrightarrow{\gamma} x$ [(7), p. 494].

En employant cette définition, on peut construire dans chaque espace (L) linéaire X, outre la convergence y donnée (γ), sa convergence (*). Cette convergence satisfait de même à toutes les conditions 1) — 7), et elle est en général plus large que la convergence (γ). Si l'on définit maintenant une topologie dans X, comme il a été fait dans le § 1, mais en employant la convergence (*), on obtient le même espace topologique. Mais, contrairement au cas de la convergence (γ), la convergence (*) coïncide avec la convergence topologique [(7), p. 497]. Donc, si la convergence (γ) définie dans X coïncide avec sa convergence (*), on obtient de X au moyen du procédé indiqué dans le § 1 un espace linéaire topologique **. Dans ce cas, nous dirons que X est un espace (LT).

§ 3. Définitions. Soit X un espace (LT). Un ensemble $M \subset X$ est borné, si quels que soient $x_n \in M$ et $\alpha_n \to 0$, on a $\alpha_n x_n \xrightarrow{\gamma} 0$ (9). Un ensemble $M \subset X$ est convexe, si les relations $x_n \in M$, $x_n \geqslant 0$ $(n = 1, 2, \ldots k)$ et $\alpha_1 + \ldots + \alpha_k = 1$ impliquent $\sum_{n=1}^{k} \alpha_n x_n \in M$.

Une métrique invariante par rapport à l'opération d'addition, c.-à-d. telle que $\rho(x, y) = \rho(x - y, 0)$, sera dite métrique (F).

Pour les espaces (LT) restent vrais les théorèmes suivants, démontrés déjà pour les espaces linéaires topologiques.

Théorème 1. *Pour qu'il soit possible de normer un espace (LT) X au sens de M. Banach* (10) *(de sorte que la convergence topologique coïncide avec la convergence suivant la norme), il faut et il suffit qu'il existe dans X un voisinage convexe et borné de l'élément zéro* (1).

Théorème 2. *Pour qu'un espace (LT) X soit métrisable (F) (de sorte que la convergence topologique coïncide avec la convergence suivant la*

* Il est à remarquer que la fermeture d'un ensemble dans l'espace topologique construit ci-dessus ne coïncide pas en général avec la somme de cet ensemble et de l'ensemble de tous ses points-limites (γ). Cf. (7).

** Nous donnons ici à la notion d'espace linéaire topologique un sens plus large que celui qui lui a été attribué par A. Kolmogoroff (1) ou J. v. Neumann (8), car nous supposons que les opérations d'addition et de multiplication ne sont continues que par rapport à la convergence des suites, et non pas au sens topologique.

distance), *il faut et il suffit qu'il existe dans* X *un système dénombrable complet de voisinages de l'élément zéro* *.

Le dernier théorème a été démontré par Garrett Birkhoff pour les groupes topologiques [11]. Les démonstrations de ces deux théorèmes pour les espaces (LT) peuvent être reproduites de la même manière comme pour les espaces linéaires topologiques, bien que la continuité de l'addition et de la multiplication dans les espaces (LT) soit plus faible.

Il est facile à voir que s'il existe dans X un voisinage borné de zéro, alors on peut extraire de tous ses voisinages un système complet dénombrable. On arrive ainsi au corollaire suivant.

Corollaire. *S'il existe dans* X *un voisinage borné de zéro, alors* X *est métrisable* (F) [2].

Dans tous les cas considérés, on peut affirmer que l'espace X est complet dans le sens métrique, s'il est complet (γ), c.-à-d. la relation $x_n - x_m \xrightarrow{\gamma} 0$ étant satisfaite pour tous les $n, m \to \infty$, on a $x_n \xrightarrow{\gamma} x$.

§ 4. Passons à la considération d'espaces (L) linéaires. Dans les cas où la convergence (γ) ne coïncide pas avec sa convergence (*), on ne peut pas la représenter comme une convergence suivant la norme ou la distance, et nous étudierons ici une métrisation d'espaces plus générale.

Définitions. Supposons que pour chaque complexe fini $K = (x_1, \ldots, x_\nu)$ d'éléments d'un espace (L) linéaire X une fonction numérique métrique $\rho[K] \geqslant 0$ est définie et qu'elle satisfait aux conditions suivantes:

I. Si $x_i = x_j$, on a

$$\rho[(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_\nu)] = \rho[(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_j, \ldots, x_\nu)] = \rho[(x_1, \ldots, x_i, \ldots, x_{j-1}, x_{j+1}, \ldots, x_\nu)].$$

II. $\rho[(x)] = 0$ est équivalent à $x = 0$.

III. $\rho[(x_1^{(1)}, \ldots, x_\nu^{(1)}, x_1^{(2)}, \ldots, x_\mu^{(2)})] \leqslant \rho[(x_1^{(1)}, \ldots, x_\nu^{(1)})] + \rho[(x_1^{(2)}, \ldots, x_\mu^{(2)})]$.

III_1. $\rho[(x_1, \ldots, x_\nu, x_{\nu+1})] \geqslant \rho[(x_1, \ldots, x_\nu)]$.

IV. $\rho[(x_1^{(1)} + x_1^{(2)}, \ldots, x_\nu^{(1)} + x_\nu^{(2)})] \leqslant \rho[(x_1^{(1)}, \ldots, x_\nu^{(1)})] + \rho[(x_1^{(2)}, \ldots, x_\nu^{(2)})]$.

Posons $x_n \xrightarrow{k} x$, si $\rho[(x_n - x, \ldots, x_m - x)] \to 0$ pour $n, m \to \infty$.

Si, d'ailleurs, la convergence (γ) dans X coïncide avec la convergence (k) ainsi définie, nous dirons que l'espace X est métrisable (KF) **.

Si, en outre, la fonction métrique $\rho[K]$ satisfait à une condition complémentaire:

V. $\rho[(\alpha x_1, \ldots, \alpha x_\nu)] = |\alpha| \cdot \rho[(x_1, \ldots, x_\nu)]$ ***, nous dirons que l'espace X est normable (K) [12].

Pour répondre à la question dans quel cas un espace (L) linéaire est métrisable (KF), nous introduirons une topologie dans l'ensemble $\mathfrak{K}$ de tous les complexes finis K d'éléments de X. D'abord, nous considérerons $\mathfrak{K}$ comme un ensemble linéaire. Dans ce but, si $K = (x_1^{(1)}, \ldots, x_\nu^{(1)})$, $K_2 = (x_1^{(2)}, \ldots, x_\mu^{(2)})$ et $\mu \geqslant \nu$, posons

$$K_1 + K_2 = K_2 + K_1 = (x_1^{(1)} + x_1^{(2)}, \ldots, x_\nu^{(1)} + x_\nu^{(2)}, x_{\nu+1}^{(2)}, \ldots, x_\mu^{(2)}).$$

* Un tel espace ne diffère des espaces du type (F) de M. Banach qu'en ce qu'il peut ne pas être complet.

** Une autre note sera consacrée à une étude particulière des espaces avec une métrique (KF).

*** Dans ce cas la condition III_1 est une conséquence des autres [13].

Définissons la multiplication comme ordinairement: si $K=(x_1, \ldots, x_\nu)$, posons $\alpha K=(\alpha x_1, \ldots, \alpha x_\nu)$. En outre, nous identifions les complexes $(x_1^{(1)}, \ldots, x_\nu^{(1)})$, et $(x_1^{(2)}, \ldots, x_\mu^{(2)})$ $(\mu \geqslant \nu)$, si $x_i^{(1)}=x_i^{(2)}$ $(i=1, 2, \ldots, \nu)$ et $x_{\nu+1}^{(2)}=\ldots=x_\mu^{(2)}=0$. Alors $\mathfrak{R}$ satisfait à tous les axiomes d'un ensemble linéaire [(10), p. 26].

Soit, maintenant, $K_n=(x_1^{(n)}, \ldots, x_{\nu_n}^{(n)}) \in \mathfrak{R}$ $(n=1, 2, \ldots)$. Posons $K_n \xrightarrow{k\gamma} 0$, si de chaque suite partielle $\{K_{n_i}\}$ on peut extraire une autre suite partielle $\{K_{n_{i_l}}\}$ telle que la suite formée des éléments de ces complexes

$$x_1^{(n_{i_1})}, \ldots, x_{\nu_{n_{i_1}}}^{(n_{i_1})}, x_1^{(n_{i_2})}, \ldots, x_{\nu_{n_{i_2}}}^{(n_{i_2})}, \ldots, x_1^{(n_{i_l})}, \ldots, x_{\nu_{n_{i_l}}}^{(n_{i_l})}, \ldots \xrightarrow{\gamma} 0.$$

Il résulte des propriétés de la convergence (γ), que la convergence $(k\gamma)$ vers zéro d'une suite $\{K_n\}$ reste intacte, si l'on remplace quelques-uns des K_n par des complexes identifiés avec ces K_n. Il est facile à vérifier que $\mathfrak{R}$ est un espace (LT).

Définition. Appelons régulière la convergence (γ) dans un espace (L) linéaire, si la relation $x_n \xrightarrow{\gamma} 0$ implique l'existence de nombres $n_i \to \infty$ et $p_i \geqslant 0$ tels qu'il est impossible d'extraire de la suite des complexes $K_i=(x_{n_i}, \ldots, x_{n_i+p_i})$ une suite partielle $\{K_{i_l}\}$ dont les éléments forment une suite

$$x_{n_{i_1}}, \ldots, x_{n_{i_1}+p_{i_1}}, \ldots, x_{n_{i_l}}, \ldots, x_{n_{i_l}+p_{i_l}}, \ldots$$

convergente (γ) vers zéro *.

On démontre à l'aide des théorèmes 1—2 les théorèmes suivants:

Théorème 3. *Pour qu'un espace (L) linéaire X soit normable (K), il faut et il suffit que la convergence (γ) soit régulière et qu'il existe dans $\mathfrak{R}$ un voisinage convexe et borné de l'élément zéro.*

Théorème 4. *Pour que l'espace (L) linéaire X soit métrisable (KF), il faut et il suffit que la convergence (γ) soit régulière et qu'il existe dans $\mathfrak{R}$ un système dénombrable complet de voisinages de zéro.*

Corollaire. *S'il existe dans $\mathfrak{R}$ un voisinage borné de zéro et si la convergence (γ) est régulière, l'espace X est métrisable (KF).*

Dans tous ces cas, si l'espace X est complet (γ), il est de même complet (K), c.-à-d. la relation $\rho[(x_n-x_p, \ldots, x_m-x_p)] \to 0$ pour $n, m, p \to \infty$ implique $x_n \xrightarrow{k} x$.

§ 5. Les théorèmes qui précèdent peuvent être facilement appliqués aux espaces semi-ordonnés de L. Kantorovitch (14). Soit X un espace semi-ordonné linéaire satisfaisant aux axiomes I—V**. Il suit de la théorie de L. Kantorovitch que si l'on entend par convergence (γ) la convergence ordinaire [convergence (o)], alors X est un espace (L) linéaire. Or, on peut considérer dans X outre la convergence (o) sa convergence (*). Désignons par X^* l'espace (LT) qu'on obtient de l'ensemble X, si l'on prend la convergence (*) pour la convergence (γ). Le théorème qui suit peut être démontré sans aucune difficulté.

Théorème 5. *Si la convergence (o) dans un espace semi-ordonné linéaire est régulière (au sens du paragraphe précédent) et si l'espace X^**

---

* Si γ-convergence coïncide avec sa (*)-convergence, elle est nécessairement régulière.

** Nous supposons que les notions principales de l'article (14) sont connues aux lecteurs.

est normable au sens de M. Banach [*métrisable* (*F*) *resp.*], *alors l'espace X est normable* (*K*) [*métrisable* (*KF*) *resp.*].

La condition du théorème est de même nécessaire.

Remarques. 1. Si un espace semi-ordonné linéaire X est métrisable (KF), la fonction métrique peut être définie de façon qu'elle remplisse la condition:

$$\rho[(x_1, \ldots, x_\nu)] = \rho[(\sup(|x_1|, \ldots, |x_\nu|))].$$

2. La régularité de la convergence (o) dans X est assurée, si X^* est métrisable (F) de façon que la condition suivante soit remplie:

$$0 \leqslant x_1 \leqslant x_2 \leqslant \ldots \leqslant x_n \leqslant \ldots \longrightarrow \infty \text{ implique } \rho(x_n, 0) \longrightarrow \infty.$$

Institut de Mathématiques.
Université de Leningrad.

Manuscrit reçu
le 26. III. 1939.

LITTÉRATURE CITÉE

[1] A. Kolmogoroff, Studia Math., **5** (1934). [2] D. H. Hyers, Bull. Am. Math. Soc., **44** (1938). [3] J. V. Wehausen, Duke M. J., **4** (1938). [4] M. Fréchet, Les espaces abstraits (1928). [5] F. Hausdorff, Mengenlehre (1927). [6] C. Kuratowski, Topologie (1933). [7] F. Hausdorff, Fund. Math., **25** (1935). [8] J. v. Neumann, Trans. Am. Math. Soc., **37** (1935). [9] S. Mazur u. W. Orlicz, Studia Math., **4** (1933). [10] S. Banach, Théorie des opérations linéaires (1932). [11] Garrett Birkhoff, Compositio Math., **3** (1936). [12] B. Vulich, C. R. Acad. Sci. URSS, IV, 311—314 (1935); Ann. of Math., **38** (1937). [13] B. Vulich, C. R. Acad. Sci. URSS, III, 109—110 (1936). [14] L. Kantorovitch, Rec. Math. de Moscou, 2 (1937).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

MATHÉMATIQUES

SUR QUELQUES THÉORÈMES D'EXISTENCE

Par P. NOVIKOFF

(Présenté par I. M. Vinogradow, de l'Académie, le 22.III.1939)

Il est bien connu qu'en mathématiques il existe des moyens pour démontrer des théorèmes d'existence sans recourir à la construction de l'objet dont on démontre l'existence. Dans plusieurs cas la démonstration de l'existence ne facilite nullement une pareille construction.

Dans le présent article nous considérons les théorèmes d'existence du type suivant: soit $F(n)$ une certaine proposition, concernant un entier positif, dont on vérifie si elle est vraie ou fausse d'une manière régulière et n'exigeant qu'un nombre fini d'opérations pour chaque n [par exemple, $F(n)$ peut être l'affirmation que n est premier, parfait, etc.]. Supposons que l'on a démontré l'existence d'un entier positif n pour lequel $F(n)$ est vrai; il est naturel de poser la question s'il est possible, dans ces conditions, de trouver des méthodes qui permettent de déterminer ce nombre tout-à-fait effectivement. Il nous semble qu'il est vraiment possible de trouver des méthodes pareilles en ne faisant que des hypothèses très peu restrictives.

Il est vrai que dans un certain calcul logique, pour lequel on n'a pas encore démontré qu'il n'est pas contradictoire, il peut arriver qu'il existe des démonstrations ne donnant aucune indication sur le nombre n, mais il est probable qu'en démontrant la non-contradiction du calcul on trouvera en même temps l'indication du nombre n.

Le présent article contient l'exposé des résultats suivants: 1) la construction d'un certain calcul logique assez puissant où le problème précédent est posé d'une manière parfaitement nette; 2) la solution affirmative du problème posé dans ce calcul.

Dans le calcul logique considéré nous avons des lettres en infinité dénombrable: $A, B, C, \ldots$ et des formules définies de la manière suivante:

1. Chaque lettre est une formule.
2. Si F est une formule, (F) l'est aussi, et inversement.
3. Si $F_1, F_2, \ldots, F_n, \ldots$ est une suite finie ou dénombrable de formules, alors $(F_1)\cdot(F_2)\ldots(F_n)\ldots$ et $(F_1)+\ldots+(F_n)+\ldots$ seront des formules *.
4. Si F est une formule, $\overline{F}$ l'est aussi (nous n'introduisons pas ici le signe d'implication; le rôle de la formule $F \rightarrow G$ sera joué par $\overline{F}+G$).

* Dans ces notations le signe du produit signifie «et» et le signe de somme — «ou bien».

Il est évident que le calcul des propositions ordinaire est une partie du calcul indiqué. Nous allons admettre des raisonnements métamathématiques quelconques sur le contenu des formules admis en mathématique intuitionniste.

Les règles pour déduire ou former des formules vraies seront comme il suit.

On considère comme formules vraies toutes les formules vraies du calcul fini classique des propositions.

1. Règle de substitution:

Si $F(A_1, A_2, \ldots, A_n, \ldots)$ est une ormule vraie, alors $F(G_1, G_2, \ldots, G_n, \ldots)$ ou l'on substitue G_n à la place de A_n est une formule vraie.

2. Si $\overline{F}+G_n$ est une suite de formules vraies, il en est de même pour $\overline{F}+(G_1, G_2, \ldots G_n, \ldots)$.

3. Si $\overline{G_n}+F$ est une suite de formules vraies, il en est de même pour $\overline{G_1+G_2+\ldots+G_n+\ldots}+F$.

4. Règle de conséquence: si F et $\overline{F}+G$ sont vraies, G l'est aussi.

5. L'expression de la forme $F_1 F_2 \ldots F_n \ldots$ et l'expression $(F_{n_{11}} F_{n_{12}} \ldots)\times$ $\times(F_{n_{21}} \ldots)\ldots$, où n_{ij} parcourt tous les entiers positifs qui sont des indices de F, sont vraies simultanément; il en est de même pour le signe $+$.

6. Dans l'expression de la forme $(F_1 F_2 \ldots F_n \ldots)$ on peut changer l'ordre des termes F_n; il en est de même pour le signe $+$.

Introduisons la notion de formule normale. Chaque lettre et sa négation seront considérées comme formules normales. D'ailleurs, si les F_n sont des formules normales et ont la forme $(F_{n1})+(F_{n2})+\ldots$ $(F_{nk})+\ldots$, alors $(F_1)(F_2)\ldots(F_n)\ldots$ sera une formule, et si F_n est de la forme $(F_{n1})(F_{n2})\ldots(F_{nk})\ldots$, alors $(F_1)+(F_2)+\ldots+(F_n)+\ldots$ sera une formule normale. Chaque formule peut être mise sous forme normale, quand on fait disparaître les parenthèses. Pour chaque formule on peut obtenir une formule normale correspondante qui lui est équivalente en ce sens que si l'une d'elles est vraie, l'autre l'est aussi.

Introduisons une nouvelle notion, celle de régularité des formules. Cette notion sera définie pour les formules normales. Une formule qui n'est pas normale sera dite régulière, si la formule normale qui lui est équivalente est régulière. Pour les formules normales la définition de régularité est la suivante. Les formules de la forme $A+\overline{A}$ sont régulières. La formule normale $F_1 F_2 \ldots F_n \ldots$ est régulière si chacune des formules F_n est régulière. L'ensemble des formules F_n sera nommé système déterminant pour la formule $F_1 F_2 \ldots F_n \ldots$ La formule normale F_1+F_2+ $+\ldots+F_n+\ldots$, où F_n est de la forme $(F_{n1} F_{n2} \ldots F_{nk} \ldots)$, est régulière, si les formules suivantes sont régulières:

$$\begin{array}{l} F_{r_1}+F_{n_{11}}+F_{n_{12}}+\ldots \\ F_{r_2}+F_{n_{21}}+F_{n_{22}}+\ldots \\ \quad\vdots \\ F_{rk}+F_{n_{k1}}+F_{n_{k2}}+\ldots \\ \quad\vdots \end{array}$$

où n_{pq} et k sont des entiers positifs arbitraires et r est un nombre bien déterminé. L'ensemble des formules indiqué sera nommé système déterminant de la formule $F_1+F_2+\ldots+F_n+\ldots$

La définition des formules régulières permet de déduire leurs propriétés par induction. En effet, pour établir que toutes les formules régulières jouissent d'une certaine propriété, il suffit de démontrer que toutes les formules de la forme $A+\overline{A}$ jouissent de cette propriété; ensuite en supposant que les formules du système déterminant jouissent de la

propriété considérée, démontrer que la formule F elle-même jouit de cette propriété.

On peut démontrer que:

Théorème I. *Toutes les formules régulières sont vraies dans le sens précédemment indiqué.*

Théorème II. *Inversement, toutes les formules vraies sont régulières.*

La première proposition est très facile à démontrer. Pour établir la proposition inverse on démontre d'abord que les formules du calcul fini des propositions sont régulières, et ensuite qu'en appliquant les règles de déduction à des formules régulières, on obtient des formules encore régulières. De ces théorèmes on déduit immédiatement que le calcul est non-contradictoire, c'est-à-dire l'impossibilité de démontrer simultanément F et $\overline{F}$. Si l'on avait une contradiction, on aurait pu en appliquant les règles de déduction démontrer une formule quelconque, par exemple A. Or, il est facile à démontrer par induction qu'aucune formule régulière ne peut être composée d'une seule lettre, donc A ne peut pas être une formule régulière, donc, en vertu du théorème II, elle ne peut être déduite par les règles du calcul.

Dans le calcul considéré, le problème précédemment posé sur les théorèmes d'existence peut être énoncé d'une manière tout-à-fait précise, à savoir: supposons que l'on a démontré la formule $F_1+F_2+\ \dots\ +F_n+\ \dots$, où chaque F_n est une formule du calcul fini. Existe-t-il un entier N tel que $F_1+F_2\ \dots\ F_N$ est une formule vraie? En vertu du théorème II cette question se résout par l'affirmative.

Supposons que l'on a démontré la formule $F_1+F_2+\ \dots\ +F_n+\ \dots$ dont on suppose qu'elle est normale. Comme elle doit être régulière, il existe un nombre r, tel que les formules

$$\begin{array}{l} F_{r_1}+F_{n_{11}}+F_{n_{12}}+\ \dots \\ \qquad\vdots \\ F_{rk_r}+F_{n_{k_r1}}+F_{n_{k_r2}}+\ \dots \end{array}$$

sont aussi régulières. Il est évident que toutes les formules du système déterminant donné sont du même type que la formule $F_1+F_2+\ \dots+F_n+\ \dots$ elle-même. L'affirmation énoncée est vraie pour les formules régulières élémentaires. Supposons qu'elle est vraie pour les formules du système déterminant, c'est-à-dire que pour chaque k il existe un nombre N_k tel que la formule $F_1+F_2+\ \dots\ +F_{Nk}$ est vraie. Dans le cas considéré k ne parcourt qu'un nombre fini de nombres de 1 à k_r. Prenons le nombre N tel que $N=\max(N_1, N_2\ \dots\ N_{k_r}, r)$, alors nous aurons évidemment que $F_1+F_2+\ \dots\ +F_N$ est une formule vraie.

Manuscrit reçu
le 23.III.1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

MECHANICS

ON THE DEFORMATIONS OF NATURALLY TWISTED BARS *

By P. RIZ

(Communicated by I. M. Vinogradow, Member of the Academy, 19. III. 1939)

In the present part of our paper we consider the torsion of a naturally twisted bar by forces distributed over the butt cross-section and statically equivalent to a couple with the torque M_0.

Observe that in the unstressed state the cross-sections of the bar are turned with respect to each other by the angle $\alpha(z)$. We confine ourselves to the case $\alpha(z) = \theta z$, and demand that the equations of the theory of elasticity should be satisfied up to the terms of the order θ^2.

We remind the reader that beside the fundamental system of co-ordinates x, y, z, whose origin is placed in the centre of gravity of one of the cross-sections, we use co-ordinate axes ξ, η, ζ, rotating with the cross-sections. These last co-ordinates are connected with the fundamental ones by the following relations:

$$\left.\begin{aligned} \xi &= x\cos\alpha - y\sin\alpha, \\ \eta &= x\sin\alpha + y\cos\alpha, \\ \zeta &= z. \end{aligned}\right\} \qquad (1)$$

Let the equation of the lateral surface of the bar in these co-ordinates be $f(\xi, \eta) = 0$.

We assume the following expressions for the displacements:

$$\left.\begin{aligned} u &= -\tau yz + \tau\theta u^{(1)}(\xi, \eta, \zeta), \\ v &= \tau xz + \tau\theta v^{(1)}(\xi, \eta, \zeta), \\ w &= \tau\varphi(\xi, \eta) + \tau\theta w^{(1)}(\xi, \eta, \zeta). \end{aligned}\right\} \qquad (2)$$

Here φ is the torsion function for the contour $f(\xi, \eta) = 0$, $\tau = \frac{M_0}{\mu T}$, T — the geometrical torsion stiffness of the corresponding prismatic bar. Then we obtain for the stresses:

$$\left.\begin{aligned} \sigma_{xx} &= \lambda\tau\theta H + \tau\theta\sigma_{xx}^{(1)}, \quad \sigma_{yy} = \lambda\tau\theta H + \tau\theta\sigma_{yy}^{(1)}, \\ \sigma_{zz} &= (\lambda + 2\mu)\tau\theta H + \tau\theta\sigma_{zz}^{(1)}, \quad \sigma_{xy} = \tau\theta\sigma_{xy}^{(1)}, \\ \sigma_{xz} &= \mu\tau(\varphi_\xi' - y) + \mu\tau\theta z\varphi_\eta' + \tau\theta\sigma_{xz}^{(1)}, \\ \sigma_{yz} &= \mu\tau(\varphi_\eta' + x) - \mu\tau\theta z\varphi_\xi' + \tau\theta\sigma_{yz}^{(1)}, \\ H &= \xi\varphi_\eta' - \eta\varphi_\xi'. \end{aligned}\right\} \qquad (3)$$

* For the first part of the present paper see this Journal, vol. XXIII, № 1 (1939).

For the unknown additional stresses we obtain the following equations of equilibrium:

$$L_1^{(1)} + (\lambda+\mu)\frac{\partial H}{\partial \xi} = 0,\ L_2^{(1)} + (\lambda+\mu)\frac{\partial H}{\partial \eta} = 0,\ L_3^{(1)} = 0. \tag{4}$$

Herein

$$L_1^{(1)} = \frac{\partial \sigma_{xx}^{(1)}}{\partial \xi} + \frac{\partial \sigma_{xy}^{(1)}}{\partial \eta} + \frac{\partial \sigma_{xz}^{(1)}}{\partial \zeta}, \ldots$$

The conditions expressing the absence of stresses on the lateral surface are:

$$\left.\begin{aligned} \sigma_{xx}^{(1)} f'_\xi + \sigma_{xy}^{(1)} f'_\eta + \lambda H f'_\xi + \mu(\varphi'_\xi - \eta)(\xi f'_\eta - \eta f'_\xi) = 0, \\ \sigma_{xy}^{(1)} f'_\xi + \sigma_{yy}^{(1)} f'_\eta + \lambda H f'_\eta + \mu(\varphi'_\eta + \xi)(\xi f'_\eta - \eta f'_\xi) = 0, \\ \sigma_{xz}^{(1)} f'_\xi + \sigma_{yz}^{(1)} f'_\eta = 0. \end{aligned}\right\} \tag{5}$$

These equations formally represent a certain problem of elastic equilibrium under the action of mass and surface forces.

The corresponding compatibility conditions have the usual form:

$$\left.\begin{aligned} \nabla^2 \sigma_{xx}^{(1)} + \frac{\partial^2 \Sigma^{(1)}}{\partial \xi^2} \cdot \frac{1}{1+\nu} = -2(\lambda+\mu)\frac{\partial^2 H}{\partial \xi^2},\ \nabla^2 \sigma_{xy}^{(1)} + \\ + \frac{\partial^2 \Sigma^{(1)}}{\partial \xi \partial \eta} \cdot \frac{1}{1+\nu} = -2(\lambda+\mu)\frac{\partial^2 H}{\partial \xi \partial \eta}, \\ \nabla^2 \sigma_{yy}^{(1)} + \frac{\partial^2 \Sigma^{(1)}}{\partial \eta^2} \cdot \frac{1}{1+\nu} = -2(\lambda+\mu)\frac{\partial^2 H}{\partial \eta^2},\ \nabla^2 \sigma_{xz}^{(1)} + \\ + \frac{\partial^2 \Sigma^{(1)}}{\partial \xi \partial \zeta} \cdot \frac{1}{1+\nu} = 0, \\ \nabla^2 \sigma_{zz}^{(1)} + \frac{\partial^2 \Sigma^{(1)}}{\partial \zeta^2} \cdot \frac{1}{1+\nu} = 0,\ \nabla^2 \sigma_{yz}^{(1)} + \frac{\partial^2 \Sigma^{(1)}}{\partial \eta \partial \zeta} \cdot \frac{1}{1+\nu} = 0, \\ \text{where } \Sigma^{(1)} = \sigma_{xx}^{(1)} + \sigma_{yy}^{(1)} + \sigma_{zz}^{(1)}. \end{aligned}\right\} \tag{6}$$

All these conditions are satisfied, if we choose the additional stresses as follows:

$$\left.\begin{aligned} &\sigma_{xz}^{(1)} = 0,\ \sigma_{yz}^{(1)} = 0, \\ &\sigma_{xx}^{(1)} = -(\lambda+\mu)H + \frac{\partial^2 F}{\partial \eta^2}, \\ &\sigma_{yy}^{(1)} = -(\lambda+\mu)H + \frac{\partial^2 F}{\partial \xi^2}, \\ &\sigma_{xy}^{(1)} = -\frac{\partial^2 F}{\partial \xi \partial \eta}, \\ &\sigma_{zz}^{(1)} = \nu \nabla^2 F - 2(\lambda+\mu)H + A\xi + B\eta + C, \end{aligned}\right\} \tag{7}$$

where F is a function satisfying the equation

$$\nabla^4 F = 0 \tag{8}$$

and the conditions on the contour

$$\left.\begin{aligned} \frac{\partial^2 F}{\partial \eta^2} f'_\xi - \frac{\partial^2 F}{\partial \xi \partial \eta} f'_\eta = \mu H f'_\xi - \mu(\varphi'_\xi - \eta)(\xi f'_\eta - \eta f'_\xi) \\ -\frac{\partial^2 F}{\partial \xi \partial \eta} f'_\xi + \frac{\partial^2 F}{\partial \eta^2} f'_\eta = \mu H f'_\eta - \mu(\varphi'_\eta + \xi)(\xi f'_\eta - \eta f'_\xi). \end{aligned}\right\} \tag{9}$$

Here are the expressions for the complete system of stresses:

$$\left.\begin{aligned}
\sigma_{xz} &= \mu\tau\,(\varphi'_\xi - \eta) + \mu\tau\theta z\,(\varphi'_\eta + \xi),\\
\sigma_{yz} &= \mu\tau\,(\varphi'_\eta + \xi) - \mu\tau\theta z\,(\varphi'_\xi - \eta),\\
\sigma_{xx} &= -\mu\tau\theta\left\{H + \frac{1}{\mu}\frac{\partial^2 F}{\partial \eta^2}\right\},\\
\sigma_{yy} &= -\mu\tau\theta\left\{H + \frac{1}{\mu}\frac{\partial^2 F}{\partial \xi^2}\right\},\\
\sigma_{xy} &= -\tau\theta\frac{\partial^2 F}{\partial\xi\partial\eta},\\
\sigma_{zz} &= \tau\theta\left\{\nu\nabla^2 F + A\xi + B\eta + C\right\}.
\end{aligned}\right\} \qquad (10)$$

The constants A, B and C may be determined from the integral relations on the butt surface expressing the statical equivalence of stresses to a couple:

$$\int\int \sigma_{zz}\,d\xi\,d\eta = \int\int \xi\sigma_{zz}\,d\xi\,d\eta = \int\int \eta\sigma_{zz}\,d\xi\,d\eta = 0. \qquad (11)$$

Observe also that the following conditions are satisfied:

$$\int\int \sigma_{xz}\,d\xi\,d\eta = \int\int \sigma_{yz}\,d\xi\,d\eta = 0,\ \int\int (x\sigma_{yz} - y\sigma_{xz})\,d\xi\,d\eta = M_0, \qquad (12)$$

and we obtain the usual expression for the geometrical torsion stiffness:

$$T = \int\int (\xi^2 + \eta^2 + \xi\varphi'_\eta - \eta\varphi'_\xi)\,d\xi\,d\eta \qquad (13)$$

(in Kirchhoff's theory of thin bars this last result is directly postulated).

For the displacements we easily obtain the following result:

$$\left.\begin{aligned}
u &= -\tau yz + \tau\theta\bar{u}\,(\xi, \eta),\\
v &= \tau xz + \tau\theta\bar{v}\,(\xi, \eta),\\
w &= \tau\varphi\,(\xi, \eta) + \tau\theta z\left(\alpha_1 x + \alpha_2 y + \alpha\right).
\end{aligned}\right\} \qquad (14)$$

The constants α, α_1, α_2 are connected with A, B, C by the following relations:

$$\alpha = \frac{C}{2\,(\lambda+\mu)},\ \alpha_1 = \frac{A}{2(\lambda+\mu)},\ \alpha_2 = \frac{B}{2(\lambda+\mu)}. \qquad (15)$$

These expressions show that the displacements are composed of: 1) the usual torsion; 2) a certain additional «plane» deformation determined by the functions $\bar{u}$ and $\bar{v}$; 3) an axial tension or compression characterized by the constant α (the compression takes place in the case, when the elastic torsion has the same sense as the natural twist); 4) a certain additional deformation of the cross-sections depending on z.

We may remark that in order to realize the considered deformation we must distribute the external forces over the butt cross-section in a certain definite manner. However, by St. Venant's principle, we may assume that in the case of a different distribution of forces the deviation from the obtained deformation will have the character of local excitations.

It may be interesting to point out a certain similitude between the deformation of torsion of a naturally twisted bar and the so-called great

deformation of torsion, for which the linear relations of the theory of elasticity cease to be valid.

For deformations of this kind, which have been considered by the author in the case of a circular cylinder (2) and by D. Panov in the case of an elliptical cylinder (3), the torsion is also accompanied by axial compression and certain additional deformations similar to the additional deformations arising at the torsion of naturally twisted bars.

In this connexion it is of interest to point out that the energy of the deformation is given in our case by

$$A = \frac{1}{2}\mu T \tau^2 + \tau \int\int \sigma_{zz} \varphi(\xi, \eta)\, d\xi\, d\eta,$$

up to terms of the order θ^2.

It is easily shown that for contours possessing a centre of symmetry the last integral is equal to zero, if the torsion axis passes through the centre of symmetry, and that, if this is not the case, this integral may be different from zero and its value depends, in general, on the choice of the axis of the natural twist and the elastic torsion.

As we have already pointed out, in the present investigation this axis was supposed to pass through the centres of gravity of the cross-sections.

Received
21. III. 1939.

REFERENCES

[1] P. Riz, C. R. Acad. Sci. URSS, XXII, No 9 (1939). [2] P. Riz, ibid., XX, No. 3—4 (1938). [3] D. Panov, ibid., XXII, No 4 (1939).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

ASTROPHYSICS

EQUATION OF THE TRANSPORT OF ENERGY BY CONVECTION CURRENTS IN STARS

By A. I. LEBEDINSKY

(Communicated by W. G. Fessenkov, Member of the Academy, 23.III.1939)

Siedentopf (1) in his theory of solar granulation, and Biermann (2) in studying the problem of convection in inner parts of stars and in their atmospheres, use the equation of the transfer of energy taken from the theory of turbulence

$$H_k = c_p \rho v l \left[\left(\frac{dT}{dz} \right)_{ad} - \frac{dT}{dz} \right] \qquad (1)$$

where: c_p is heat capacity under constant pressure, ρ — density, $\frac{dT}{dz}$ — temperature gradient, v — velocity, l — «length of the mixing way» (Mischungsweg) of convection currents. The star is assumed to be stationary, and the net flux of energy ($H_0 = H_k + H_R$) equal to the sum of fluxes transferred by convection and radiation. The method of demonstration of equation (1) in the theory of turbulence is true only in the case, when accelerations of the convection currents can be considered as independent on the quantity of the transferred heat energy (3). This is approximately true when there is to be considered the transfer of heat by turbulent eddies occurring in the flux of liquid or gas; and this becomes wrong in the case of a thermal convection stimulated by the fact of instability. For the case of a thermal convection the writer gives below an equation more accurate than (1); equation (1) may be obtained from the former in result of an averaging.

Isothermal and isobaric surfaces are to be considered as parallel planes. Axis z of the right-angled system of co-ordinates (x, y, z) is set perpendicularly to them. The increase of z proceeds from the centre of the star. The components of the velocity vector $\vec{U}$ are designated as U_x, U_y and U_z, the temperature, pressure and density in the given point as T, p and ρ respectively. Letters with a line ($\overline{U}_x$, $\overline{T}$, etc.) are used to designate mean values of U_x, T, etc. at the height z; as regards the mean values of items in ascending currents, i. e. in points where $U_z > 0$, they are designated as $\overline{\overline{U}_x}$, $\overline{\overline{T}}$, etc. $\overline{U}_x$, $\overline{T}$, $\overline{\overline{U}_x}$, $\overline{\overline{T}}$ are, in general, functions of height z and time t; in the case of a stationary atmosphere, they are functions of z only.

It is assumed that there is no circulation in the star and $\overline{U_x \rho} = \overline{U_y \rho} = \overline{U_z \rho} = 0$. The gas is ideal, and thus $p = \frac{R}{\mu} T\rho$ and $c_p - c_v = \frac{R}{\mu}$. Neglecting the radiation pressure as compared with gas pressure, Eyler's equation can be written in the form

$$\text{grad } p + \rho \frac{D\vec{U}}{Dt} + \vec{X}\rho = 0; \; X_x = X_y = 0, \; X_z = g. \qquad (2)$$

Assuming the coefficient of absorption k to be independent of the frequency of radiation, the equation of the law of energy conservation can be written as follows

$$k\int Id\omega - 4\pi kB = J = c_p\frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt}, \tag{3}$$

where $\int Id\omega$ — energy density of radiation, and $B=\frac{\sigma}{\pi}T^4$ — the black body radiation. The substitution of $\frac{\partial p}{\partial x}$, $\frac{\partial p}{\partial y}$ and $\frac{\partial p}{\partial z}$ in (3) with the aid of (2) leads to the formation of a rather cumbersome expression. The latter is to be simplified by the use of the equation of continuity $\frac{\partial \rho}{\partial t}+\operatorname{div}(\rho\vec{U})=0$; its form will be as follows

$$\frac{\partial}{dt}[\rho(c_v T+M)]+g\rho U_z+\operatorname{div}\{\rho\vec{U}[c_p(\overline{T}+\delta)+M]\}=\rho J. \tag{4}$$

In the last member of the left-hand side instead of T there is put $\overline{T}+\delta=T$. The designation $M=\frac{1}{2}(U_x^2+U_y^2+U_z^2)$ is introduced as well.

Further, both sides are multiplied by $dxdy$ and the integration with the aid of two means is carried out — for the first time, in taking the integral all over a sufficiently large section of a horizontal plane, and, for the second, restricting the same operation but to that part of this section where $U_z>0$, i. e. inside the ascending currents. Having divided each of the integrals by the area, over which it has been spread, one obtains two equations:

$$\frac{\partial}{\partial t}\overline{[\rho(c_v T+M)]}-c_p\overline{T}\frac{\partial\overline{\rho}}{\partial t}+\frac{\partial}{\partial z}\overline{[\rho U_z(c_p\delta+M)]}=\overline{\rho J} \tag{5}$$

$$\overline{\frac{\partial}{\partial t}[\rho(c_v T+M)]}-c_p\overline{T}\frac{\partial\overline{\rho}}{\partial t}+\overline{\rho U_z\left[g+c_p\frac{\partial\overline{T}}{\partial z}\right]}+$$
$$+\operatorname{div}\overline{[\vec{U}_{hor}\rho(c_p\delta+M)]}+\frac{\partial}{\partial z}\overline{[\rho U_z(c_p\delta+M)]}=\overline{\rho J}. \tag{6}$$

The horizontal vector with components (U_x, U_y) is designated as $\overrightarrow{U_{hor}}$. In (5) the signs of differentiation are put before the expression under the line of averaging, the limits of integration being independent of z and t. The relation of the area, in which $U_z>0$, to that of items, in which $U_z<0$, is designated as φ. The last term of the left-hand side (6) can be substituted with the expression $\frac{1+\varphi}{\varphi}\frac{\partial}{\partial z}\left[\frac{\varphi}{1+\varphi}\overline{U_z\rho(c_p\delta+M)}\right]$, since the limits of integration are represented by the lines $U_z=0$.

It is assumed, primo, that the atmosphere is stationary and, thus, all the mean values depend only on z; secundo, that $H_0=H_k+H_R$ and, tertio, that the star is not subject to the outflow of matter; consequently, with the increase of z, the vector $\overrightarrow{\rho U}$ tends to zero. Integrating (5) from z to ∞ one obtains

$$\overline{\rho U_z(c_p\delta+M)}=-\int_z^\infty\overline{\rho J}d\xi=H_k. \tag{5a}$$

Equation (1) is a particular case of (6). In order to demonstrate it, six simplifying assumptions may be introduced: 1) Mean values of derivatives following t can be neglected;

2) $\varphi = \text{const}$; 3) $\overline{\rho U_z \delta} \approx \overline{\rho U_z}\,\overline{\delta}$; $\overline{\rho U_z \delta} \approx \overline{\varphi\rho U_z}\,\overline{\delta}$;

4) $c_p \frac{\partial}{\partial z}(\rho U_z \delta) \gg \operatorname{div}[\rho \vec{U}_{hor}(c_p\delta + M)]$;

5) $c_p \overline{\rho U_z \delta} \gg \overline{\rho U_z M}$; $c_p \overline{\rho U_z \delta} \gg \overline{\rho U_z M}$; 6) $\overline{\rho J} \approx \overline{\rho}\,\overline{J}$.

Assumptions 3) and 4) are appropriate for the case when surfaces $\delta = 0$ can be considered as coinciding with surfaces $U_z = 0$; assumption 6) becomes suitable when the optical thickness of currents is considerably greater than unity, and the difference between the mean values of flux of radiation in ascending and descending currents is small as compared with H_R. If, on the contrary, the optical thickness of currents is considerably smaller than unity, then $\overline{J} \approx \overline{J} + 4\pi k(\overline{B} - \overline{B})$.

In virtue of assumptions 3) and 5), instead of equation (5a), we obtain:

$$H_k = \varphi c_p \overline{\rho}\,\overline{U_z}\,\overline{\delta} \qquad (7)$$

and then, in utilizing all the assumptions, instead of (6)

$$\overline{\rho}\,\overline{U_z}\left[g + c_p \frac{d\overline{T}}{dz}\right] + c_p \frac{d}{dz}\overline{\rho}\,\overline{U_z}\,\overline{\delta} = \overline{\rho}\,\overline{J} = \frac{dH_k}{dz}. \qquad (8)$$

The integration of (8) with regard to (7) gives thus

$$c_p \int_{z_0}^{z} \overline{\rho}\,\overline{U_z}\left[\left(\frac{dT}{dz}\right)_{ad} - \frac{d\overline{T}}{dz}\right] d\xi = c_p \overline{\rho}\,\overline{U_z}\,\overline{\delta} - H_k = \left(\frac{1}{\varphi} - 1\right) H_k, \qquad (9)$$

where z_0 — height of the lower limit of the convection layer. If this limit is lacking, then, instead of the right-hand integral, one has to write the integral of the same function from ∞ to z. As in the instable layer the subintegral function is a positive one and in the case when z_0 coincides with the lower limit of the instable layer, φ must be smaller than unity.

Equation (1) is obtained if, instead of the differential equation (9), there is used the algebraic relation between the values of functions to be met with in (9) which are averaged according to the height within the instable layer. Thus, «the length of the mixing way» l will correspond to the thickness of the layer multiplied by the dimensionless factor proportional to $\frac{\varphi}{1-\varphi}$.

In the above-mentioned works it was stated that in the case of the convection the temperature gradient near the surface of the star is determined by conditions of the radiative equilibrium, while in the deep layers it becomes practically adiabatic. These conclusions can be obtained from (5) and (6); consequently, their accuracy is quite obvious.

The assumptions, introduced by the writer, when carefully examined, become considerably more natural and demonstrative than at the first sight; equation (9) is, however, a rather rough approximation.

A detailed discussion of the equations given in this paper is to be published in the «Annals» of the Leningrad State University, Series of Astronomy.

Astronomic Observatory.
Leningrad State University.

Received
23.III.1939.

REFERENCES

[1] Siedentopf, Astron. Nachr., **247**, S. 297; **249**, S. 51, **255**, S. 157. [2] Biermann, Astron. Nachr., **264**, S. 361; ZS. f. Astrophys., **5**, S. 117. [3] Келлер, Изв. Гл. геофиз. обсерват., № 4 (1931).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

PHYSICS

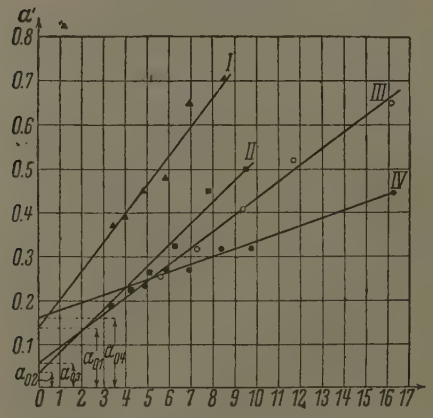
SUPERSONIC ABSORPTION IN HELIUM

By P. KRASNOOSHKIN and E. PUMPER

(Communicated by L. I. Mandelstam, Member of the Academy, 22. III. 1939)

The absorption coefficient of a sound wave in the case of a monoatomic gas coincides, as it seems, with the theoretical value given by Stokes and Kirchhoff. This has been approximately verified for argon by the measurements of Dadaian and Pumper (1). There is sufficient ground to expect that Stokes's and Kirchhoff's theory must also give the true value of the absorption constant in other monoatomic gases.

Dependence of full extinction on helium pressure.



p—pressure in atmosphere
α'—full extinction
α_0—dispersion coefficient
α—absorption coefficient
I. $\alpha\lambda^2=7.25\cdot10^{-4}$; $f=951.6$ kH
II. $\alpha\lambda^2=7.1\cdot10^{-4}$; $f=809.5$ kH
III. $\alpha\lambda^2=6.9\cdot10^{-4}$; $f=724.4$ kH
IV. $\alpha\lambda^2=5.1\cdot10^{-4}$; $f=586.12$ kH

At the same time, the experimental results for helium and neon published by Abello (2), Curtis (3), Itterbeek and Thys (4) do not agree with the theoretical values (being at least three times larger). If the experimental results published previously on this subject were true, it would be necessary to revise the principle physical conceptions on sound propagation in monoatomic gases.

In order to make this question clear, we have made a series of measurements of the supersonic absorption in helium at frequency 586.12 kH per second which lies very near that used by Itterbeek, and also at frequencies 724.54, 809.5 and 951.59 kH per second. On account of the importance of this question, we thought advisable to publish some preliminary results of our experiments, though their precision is relatively small (20—30%), for they lead us to conclude that the results obtained by the above-mentioned authors are erroneous and there is no ground for a radical revision of the theory.

As was pointed out in the above-mentioned paper by Dadaian and Pumper, all authors who used methods of absorption measurements based upon the application of a plane wave, controlled the uniformity of the sound field

in a very primitive way. Practically, in most cases the supersonic field of a piezo-quartz differs from a plane one and this non-uniformity of the field cannot be overlooked without risk of a considerable distortion in the results.

From our point of view, exceedingly high values of absorption in helium obtained by Itterbeek, Curtis and others can be explained by this fact.

The results published below were obtained by means of the methods described in the above-mentioned paper by Dadaian and Pumper, taking into account the influence of the non-uniformity of the field in first approximation, by means of an additional term α_0 in the absorption coefficient: α_0 does not depend on pressure. Then, the experimentally obtained curves, in which the total amplitude decrement $\alpha' = f(\omega, p, \alpha_0)$ is plotted against the reciprocal pressure $1/p$, can be exprerssed in the form $\alpha' = \varphi(\omega, p) + \alpha_0$, where $\varphi(\omega, p) = \alpha$ is the absorption coefficient of the gas referred to amplitude. According to Stokes and Kirchhoff, $\alpha = k\frac{\omega^2}{p}$ which was proved experimentally in the case of air and argon.

Fig. 1 shows the data obtained by us for helium. The following table gives the values $\alpha\lambda^2$ calculated from these data and referred to normal pressure.

Freq. in kH	951.6	809.5	724.54	586.12
$\alpha\lambda^2 \times 10^{-4}$	7.25	7.10	6.90	5.10
$\frac{\Delta\alpha\lambda^2}{\alpha_s\lambda^2}$, %	+43.5	+40.5	+36.5	+ 0.9

$\alpha_s\lambda^2$ is the theoretical value obtained by Stokes and Kirchhoff equal to $5.05 \cdot 10^{-4}$. From the fact that the first three values of $\alpha\lambda^2$ in the table nearly agree in magnitude, we cannot draw any conclusion because the number of investigated frequencies is too small.

We suppose that the discrepancy between theory and experiment (about 40—50%) may be due to an error in the extrapolation of the experimentally obtained straight lines and to the fact that the first approximation, allowance being made for the non-uniformity of the field, is not sufficient in helium, owing to the relatively long wave length of sound in this gas. Taking into consideration all these circumstances, we think it possible to come to the following conclusion.

The results obtained by the above-mentioned authors are erroneous. The deviations of our results from the theoretical values do not exceed 40—50% and lie within the limits of precision of the method of measurement. Thus, there is now no ground for a revision of the theory.

Laboratory of Oscillation.
Institute of Physics.
Moscow State University.

Received
26. III. 1939.

REFERENCES

[1] Dadaian a. Pumper, C. R. Acad. Sci. URSS, XX, № 7—8 (1938). [2] Abello, Phys. Rev., **31**, 1089 (1928). [3] Curtis, Phys. Rev., **46**, 811 (1934). [4] Itterbeek a. Thys, Physica, **5**, № 9, 889—897 (1938). [5] Pumper, Phys. ZS. d. Sowjetunion, 8, H. 3, 300—310 (1935).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

PHYSICS

SHAPE OF THE BETA-SPECTRUM IN THE CASE OF THE FORBIDDEN TRANSITIONS

By V. BERESTECKI

(*Communicated by A. F. Joffe, Member of the Academy, 16. III. 1939*)

The Fermi (1) theory of beta-decay with an expression for the interaction of heavy particles with a light particle field, modified according to Uhlenbeck and Konopinski (2), gives a shape of the beta-spectrum which is in good agreement with the experimental data (3) (except, however, spectral region near the upper limit). It should be noted that experimental evidence was compared with the theoretical spectrum calculated for the case of «allowed» transitions, i. e. for transitions where the angular momentum of the disintegrating nucleus does not vary. (Both in this case and farther on we have in view the selection rules which correspond to the vectorial character of the light particle field. As to the selection rules corresponding to tensor fields the position is the same). In the present paper is investigated the shape of the beta-spectrum in the case of «forbidden» transitions, i. e. such where the momentum of the disintegrating nucleus varies. Using the Fermi expression for interaction, it may be seen that the shape of the «forbidden» beta-spectrum cannot differ considerably from that of the «allowed» spectrum (4) owing to the full symmetry of this interaction expression with respect to the electron and the neutrino.

The Uhlenbeck-Konopinski expression of interaction is, however, characterized by asymmetry with respect to the electron and the neutrino. This asymmetry leads to the asymmetry of the beta-spectrum as experimentally observed, but at the same time it produces an essential difference in the shapes of the spectra for the allowed and the forbidden cases, which does not agree with experimental evidence.

Calculation Scheme. The Uhlenbeck-Konopinski theory gives for the shape of the beta-spectrum the following expression:

$$P(w)\,dw = 2\pi \frac{mc^2}{h} G^2 \sum \left| \int (\Phi^* \alpha_i \Psi)\left(\varphi^* \beta \frac{\partial}{\partial x_i} \chi\right) d\tau \right|^2 dw, \qquad (1)$$

where $P(w)\,dw$ is the probability of electron ejection with an energy from w to $w + dw$; Ψ — the wave function of the initial nucleus; Φ — the wave function of the final state of the nucleus which differs from the initial one by the fact that one definite neutron has been transformed into a proton having emitted an electron and an anti-neutrino of an

aggregate energy of w_0. The count of the number of neutrons in the nucleus, their identity and also the indiscernibility of neutrons and protons in the nucleus lead to a numerical factor which is of no interest to us[5]; $\alpha(\vec{\alpha}_1, \alpha_2, \alpha_3)$ are the Dirac matrices acting on the variables of that neutron which is transformed into a proton; the summing up from 1 to 4 is carried out over i; $\alpha_4 = -i$; φ is the wave function of the ejected electron; χ — the wave function of the neutrinos absorbed from the negative energy level, normalized for the unitary energy interval; β — the Dirac matrix, acting on the neutrino variables; $x_4 = ict$; the summing up is carried out over different states of the electron having the energy of w; G is the dimensionless universal constant.

The fact that the dimensions of the nucleus are small as compared with the wave length of the emitted particles is very essential and allows to expand the wave functions of light particles with respect to $\frac{r}{\lambda}$, confining oneself to the first term being not equal to zero.

Moreover, a non-relativistic approximation may also be used for the wave functions of the nucleus. It is easy to show that in this case, as well as in the case of one particle, it is sufficient to treat in (1) the four-rows Dirac functions Ψ and Φ as the two-rows Pauli functions and to substitute $-\frac{\vec{v}}{c}$ for $\vec{\alpha}$, where $\vec{v}$ is the velocity operator; $\vec{v} = -i\frac{w_0}{\hbar}\vec{r}$ (w_0 is the difference between the nuclear energies in the initial and the final states). It may be seen that, for all the terms of (1) to be of the same order, it is necessary, in expanding the terms containing $\vec{\alpha}$-components, to leave in the expansion of $\varphi^* \beta \frac{\partial \chi}{\partial x_i}$ the values which in r are of an order one unit less than in terms containing α_4.

We consider the light particles in a state having a definite angular momentum. Then the wave function is the product of the angular part by the radial part[6]:

$$\Psi_1 = i\sqrt{\frac{l-m+\frac{3}{2}}{2l+3}}\, Y_{l+1,m-\frac{1}{2}}(\vartheta,\varphi)\, f_k(r)$$

$$\Psi_2 = i\sqrt{\frac{l+m+\frac{3}{2}}{2l+3}}\, Y_{l+1,m+\frac{1}{2}}(\vartheta,\varphi)\, f_k(r)$$

$$\Psi_3 = \sqrt{\frac{l+m+\frac{1}{2}}{2l+1}}\, Y_{l,m-\frac{1}{2}}(\vartheta,\varphi)\, g_k(r)$$

$$\Psi_4 = -\sqrt{\frac{l-m+\frac{1}{2}}{2l+1}}\, Y_{l,m+\frac{1}{2}}(\vartheta,\varphi)\, g_k(r);$$

l and m are the azimuth and the magnetic quantum numbers, Y — the spherical function; $k = -l-1$ at $j = l+\frac{1}{2}$, $k = l$ at $j = l-\frac{1}{2}$; j is the internal quantum number. For the radial part of the electron-function we may use the solution of the Dirac equation for the conti-

nuous spectrum in the Coulomb field (7). For the neutrino function (no field is present, mass = 0), the radial functions are

$$g_k = \frac{\varepsilon}{\sqrt{2}} \frac{I_{\left|k+\frac{1}{2}\right|}(\varepsilon r)}{\sqrt{\varepsilon r}}$$

$$f_k = \frac{\varepsilon}{\sqrt{2}} \frac{I_{\left|k-\frac{1}{2}\right|}(\varepsilon r)}{\sqrt{\varepsilon r}},$$

where ε is the neutrino energy im mc^2 units, r — in $\frac{\hbar}{mc}$ units, m — the electron mass and I — the Bessel function.

If the expansion of the value of $\varphi^* \beta \frac{\partial \chi}{\partial t}$ begins with the zero power of r, then (1) will depend on the nuclear states through the value of $M = \int \Psi^* \Phi d\tau$ which defines the selection rules $\Delta j = 0$ (allowed transition). For this purpose we must use a combination of light particles in the following states:

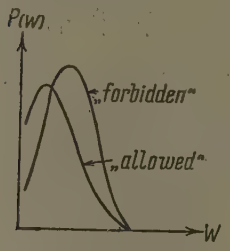


Fig. 1.

Spectroscopic symbols	
Electron	Neutrino
$S_{\frac{1}{2}}$	$S_{\frac{1}{2}}$
$P_{\frac{1}{2}}$	$P_{\frac{1}{2}}$

For the forbidden transitions, for which (1) contains $D = \int \Psi^* x \Phi d\tau$, it is necessary to use those states of light particles, for which the expansion of $\varphi^* \beta \frac{\partial \chi}{\partial t}$ begins with the first power of r (and that of $\varphi^* \beta \frac{\partial \chi}{\partial x_k}$ begins with the zero power). Such are the following combinations:

Spectroscopic symbols	
Electron	Neutrino
$S_{\frac{1}{2}}$	$P_{\frac{1}{2}}$
$P_{\frac{1}{2}}$	$S_{\frac{1}{2}}$
$S_{\frac{1}{2}}$	$P_{\frac{3}{2}}$
$P_{\frac{1}{2}}$	$d_{\frac{3}{2}}$
$P_{\frac{3}{2}}$	$S_{\frac{1}{2}}$
$d_{\frac{3}{2}}$	$P_{\frac{1}{2}}$

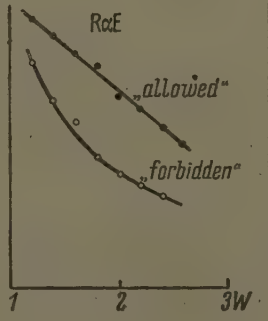


Fig. 2.

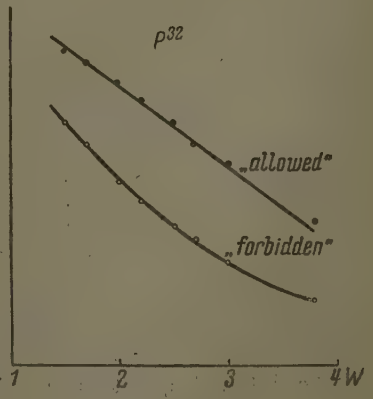


Fig. 3.

Substituting the expansions of wave functions of light particles of the above states in (1), we obtain the following equation (the electron energy w and the upper limit w_0 are given in mc^2 units):

$$P(w) = \frac{4}{9\pi^3}\frac{mc^2}{\hbar} G^2 |D|^2 (w_0 - w)^4 w\sqrt{w^2-1}\left\{F_1(Z,w)\left(3w^2-\frac{3}{2}\right)+\right.$$
$$\left.+F_2(Z,w)(w^2-1)\right\}, \qquad (2)$$

where

$$F_1(Z,w) = \frac{e^{\frac{\pi\alpha w}{p}}}{|\Gamma(1+2\sqrt{1-\alpha^2})|^2}\left|\Gamma\left(\sqrt{1-\alpha^2}+i\alpha\frac{w}{p}\right)\right|^2 (2p\rho)^{2(\sqrt{1-\alpha^2}-1)}$$

$$F_2(Z,w) = \frac{e^{\frac{\pi\alpha w}{p}}}{|\Gamma(1+\sqrt{4-\alpha^2})|^2}\left|\Gamma\left(\sqrt{4-\alpha^2}+i\alpha\frac{w}{p}\right)\right|^2 (2p\rho)^{2(\sqrt{4-\alpha^2}-2)}.$$

Z is the nuclear charge; $\alpha = \frac{Ze^2}{\hbar c}$; $p = \sqrt{w^2-1}$; ρ is the nuclear radius in $\frac{\hbar}{mc}$ units. F_1 corresponds to electron state with $j = \frac{1}{2}$, $F_2 - j = \frac{3}{2}$. For small Z's (2) is converted into

$$P(w) = \frac{5}{18}\frac{mc^2}{\hbar} G^2 |D|^2 (w_0 - w)^4 w\sqrt{w^2-1}\left(\frac{8}{5}w^2-1\right). \qquad (3)$$

Comparison with the Experiment. Fig. 1 shows the theoretical spectra of the allowed and the forbidden type of a light element with $w_0 = 4mc^2$. The shape of the forbidden spectrum differs essentially from that of the allowed type. The standard nature of the beta-spectrum of all the light elements is, however, a characteristic experimental fact. For comparison with experiment we shall use Alichanian and Zavelski's ([8]) data for RaE and those of Lyman ([9]) for P^{32}, elements, the disintegration of which is apparently of the forbidden type. The results are given in Figs. 2 and 3. On the abscissae is plotted the electron energy, and on the ordinates the solution of equation (2) with respect to $(w_0 - w)$. The experimental points do not lie on a straight line, which points to the lack of agreement with theory. For the sake of comparison the same figures show analogous curves plotted from the allowed spectrum equation.

Capture of the K-electron. The calculation of the K-electron capture by the nucleus may be carried out by means of the same scheme. In this case, the K-electron functions of the electron must be substituted in the expression (1) as the electron wave functions.

Let us juxtapose the ratio of probabilities of K-electron capture and of positron disintegration for forbidden transitions with those for the allowed transition which have been examined by Moller ([10]) (only the case of light nuclei will be investigated here).

The total probability of disintegration for allowed transitions is given by the equation

$$P_\beta^a = \frac{G^2}{2\pi^3}|M|^2 F_\beta^a$$

where

$$F_\beta^a = \int_1^{w_0}(w_0 - w)^4 w\sqrt{w^2-1}\,dw =$$

$$= \left(\frac{w_0^3}{2}+\frac{w_0}{4}\right)\ln(p_0+w_0) + \frac{p_0^7}{3\cdot 5\cdot 7} - \frac{p_0^5}{3\cdot 5} - \frac{3p_0^3}{4} - \frac{3p_0}{4}; \quad (p_0 = \sqrt{w_0^2-1})$$

and the probability of K-electron capture (the energy of K-electron being taken to be approximately mc^2) is given by

$$P_k^a = \frac{2G^2}{\pi^2}|M|^2 (w_0+1)^4 F_k$$

where

$$F_k = \frac{(Z\alpha)^2}{\Gamma(3+2s)}\left(\frac{2\rho}{a_0}\right)^{2s} e^{-\frac{2\rho}{a_0}},$$

Z is the nuclear charge; $\alpha = \frac{e^2}{\hbar c}$; $s = \sqrt{1-Z^2\alpha^2} - 1$; ρ — the radius of the nucleus in $\frac{\hbar}{mc}$ units; $a_0 = \frac{1}{Z\alpha}$.

For forbidden transitions the probability of disintegration will be

$$P_\beta^f = \frac{!5G^2}{18\pi^3} G^2 |D|^2 F_\beta^f$$

where

$$F_\beta^f = \int_1^{w_0} (w_0 - w)^4 w \sqrt{w^2-1}\left(\frac{8}{5}w^2 - 1\right) dw =$$

$$= \left(\frac{w_0^3}{4} + \frac{5w_0}{12}\right) \ln(p_0 + w_0) + \frac{P_0^9}{2\cdot 5\cdot 7\cdot 9} + \frac{P_0^7}{3\cdot 4\cdot 5\cdot 7} - \frac{P_0^5}{15\cdot 16} - \frac{P_0^3}{3\cdot 32} - \frac{13P_0}{12}$$

and the probability of capture

$$P_k^f = \frac{2G^2}{3\pi^2} |D|^2 (w_0+1)^4 F_k.$$

The ratio of capture and disintegration probabilities for a given Z is of the same order both for the allowed and the forbidden transitions, but it is slightly altered for the forbidden transitions in favour of disintegration:

$$\frac{P_k^f}{P_\beta^f} : \frac{P_k^a}{P_\beta^a} = \frac{3F_\beta^a}{F_\beta^f}.$$

The ratio of $\frac{F_\beta^a}{F_\beta^f}$ for $w_0 = 4.12$ will be 0.15.

It is so far impossible to compare the ratio of disintegration and capture probabilites with experimental evidence, since the K-electron capture and the positron disintegration of one and the same element have not yet been observed.

Note. Hoyll's paper [11] on the calculation of beta-spectrum at forbidden transitions contains an error of calculation which alters essentially the results obtained. The origin of this error is easy to trace by using in the calculations plane waves as wave functions of light particles, and expanding them into a series. In addition to factors of the $e^{\frac{i}{\hbar}\vec{pr}}$ types. these functions contain factors arranged in four-number columns. They give for the probability expression a factor which, upon summing up over the spin states, comes to a diagonal sum of a certain Dirac combination of matrices. That factor had been overlooked by Hoyll.

The writer wishes to express his thanks to A. Alichanian for the suggestion of the present work and for the interest he took in its performance. He wishes also to thank Prof. I. Tamm for his valuable advices.

Physico-Technical Institute.
Leningrad.

Received
21. III. 1939.

REFERENCES

[1] E. Fermi, ZS. f. Phys., **88**, 161 (1934). [2] E. J. Konopinski u. G. Uhlenbeck, Phys Rev., **48**, 7 (1935). [3] Alichanian, C. R. Acad. Sci. URSS, sér. phys., № 1—2 (1938). [4] K. L. Dolecek, Phys. Rev., **48**, 13 (1935). [5] L. W. Nordheim a. F. L. Yost, Phys. Rev., **51**, 942 (1937). [6] Bethe, Handbuch d. Phys., XXIV, 312. [7] M. E. Rose, Phys. Rev., **51**, 484 (1937). [8] A. Alichanian a. Zavelski, C. R. Acad. Sci. URSS, XVIII, 463 (1937). [9] E. M. Lyman, Phys. Rev., **51**, 1 (1937). [10] Moller, Sow. Phys., **11**, 9 (1937). [11] F. Hoyll, Proc. Roy. Soc., **166**, 249 (1937).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

PHYSICS

HEAT TREATMENT OF FERROMAGNETIC POLYCRYSTALS IN A MAGNETIC FIELD

By J. S. SHUR

(Communicated by P. L. Kapitza, Member of the Academy, 25.III.1939)

1. It is well known that the magnetic properties of ferromagnetic substances become sharply changed if these substances are slowly cooled in a magnetic field beginning with the Curie point (1). Ferromagnetic substances acquire then improved magnetic properties in the direction in which the magnetic field was oriented during cooling; while all at the same time in the perpendicular direction these properties remain unchanged or even become worsened. Thus, for instance, for perminvar from $\mu_{max} = 2000$ after heat treatment in the direction in which the field acted, we obtain $\mu_{max} = 190\,000$ and in the perpendicular direction $\mu_{max} = 1250$ (2).

Bozorth(3) gave the following explanation to this phenomenon. In the cooling of a ferromagnetic substance, after passage through the Curie point, regions of spontaneous magnetization are set up in it which are oriented in the direction of the axes of the easiest magnetization. If the Curie point of the given ferromagnetic substance be considerably higher than the temperature boundary of its elasticity, then the magnetostrictional stresses, apt to arise concomitantly with spontaneous magnetization, will with sufficiently slow cooling produce plastic deformation in the material, in result of which from among the axes of easy magnetization one, namely that along which the magnetization of the given region is directed, becomes the easiest axis. If cooling takes place in the absence of an external field, these «easiest» axes will be distributed in a random way. But if during cooling a field was applied, those very axes of «easy» magnetization (for each region) become «the easiest» which form the least angle with the direction of the field applied. The thus set up «texture of the fields of spontaneous magnetization» is responsible for the formation of the anisotropy of the magnetic properties of a polycrystalline ferromagnetic substance that has undergone heat treatment in a magnetic field. In such material the process of inversion will take place with much greater ease and will run in the direction in which the magnetic field was directed during cooling—a fact which becomes manifest in a considerable increase of magnetic permeability. The magnitude of the effect of annealing in a magnetic field will, on the whole, be determined by the mutual relation between the magnetic energy of anisotropy and the energy of magnetostriction (4).

However, Bozorth confines himself to a consideration of inversion processes alone. At the same time, as matters stand in reality when we deal

with an actual ferromagnetic substance, we find that after it has been cooled in a magnetic field and upon it becoming magnetized (after cooling), there will take place much more complex phenomena occasioned by rotation processes which Bozorth ignores.

2. As is well known, at high temperatures, e. g. 500—600° for iron, the crystalline anisotropy of magnetic properties is apt to disappear almost completely and the saturation of the ferromagnetic substance is attained at much lower fields ([5]). Therefore, with high temperature, even a very weak field orients all spins in its own direction, and it may be said that the whole ferromagnetic substance has to be viewed as but one gigantic region. It is only with a considerable lowering of temperature that the crystalline anisotropy of magnetic properties begins to become manifest.

If the applied field is small, then, at a certain temperature, we shall observe the disintegration of our gigantic region and new regions will be liable to be set up, oriented along those axes of easy magnetization that will be located closest of all to the direction of the field applied. At the same time, in the case of a sufficiently strong field all spins will with all temperatures remain directed exactly along the field. If we take a ferromagnetic substance whose Curie point is high enough, and if in the case in hand magnetostriction does not change the sign in a transition from the Curie point to room temperature, then we shall be apt to expect that, depending on the magnitude of the magnetic field H applied during annealing, curves of magnetization assuming different forms will be obtained with room temperature.

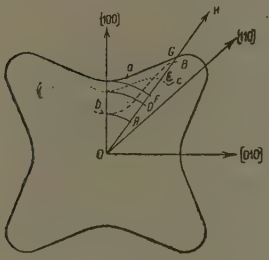


Fig. 1.

Let us dwell at somewhat greater length upon the following three cases: (a) cooling of ferromagnetic substance with $H=0$, (b) heat treatment in field H_1 which at room temperature is only apt to cause a shift of boundaries between regions and, lastly, (c) heat treatment in field H_2 which brings the whole of the ferromagnetic substance to saturation point under all temperatures. Now, it is only in case (a) that the process will take place according to Bozorth's scheme described above.

In case (b) at a certain temperature there will arise spontaneous regions oriented along those lines of easy magnetization which form the least angle with the direction of field H_1. Magnetostrictional stresses in these directions will on the whole be removed owing to plastic flow at high temperature, which will involve «the texture of spontaneous regions» being retained at room temperatures. In turn, this should lead to an enhancement of the inversion part of the magnetization curve which is due both to an increase of «antiparallel neighbourhoods» and to a more favourable location of these neighbourhoods relatively to the field adjoining them during measurements. On the other hand, that section of the magnetization curve which is caused by the rotation process should be less steep than in case (a), since more energy will be expended on these processes than in case (a), in so far as the initial position of the regions is to be found (on the average) at a deeper energy minimum than in case (a).

In case (c) during the whole time of the process of cooling our ferromagnetic substance will remain saturated in the direction of field H_2. Therefore, plastic deformation will make the direction parallel H_2 easier than in cases (a) and (b). The axes of easy magnetization forming the least angles with direction H_2 will, with such heat treatment, become somewhat more «easy» than the others, yet not to such an extent as in case (b); on account of this,

inversion processes for fields parallel to H_2 will here run their course with greater ease than with (a), but with greater difficulty than with (b). This will become quite clear from a consideration of the following simplified scheme.

Let us take a crystallite whose cube crystal face is parallel to the applied field. The dependence of free magnetic energy of anisotropy in this crystallite upon the orientation of spins may be represented by the curves of Fig. 1. This dependence can be strictly symmetrical relatively to both axes [100] and [010] only in the case, if the distribution of regions of spontaneous magnetization in this crystallite was evenly disordered along both these axes during the whole period of cooling. This condition is fully realized in case cooling takes place in a total absence of an external magnetic field. Curve *a* in Fig. 1 corresponds to this case. The field acting during the period of cooling involves asymmetry of the energy curve reflecting «the texture of the spontaneous field obtained». The different magnitude of this field will, as already shown, give curves *b* and *c*: the qualitative relationship between the energies required in the processes of inversion and in cases (a), (b) and (c) has already been discussed.

For the process of rotation in case (b) it will be necessary to expend an energy measured by section *AB* (Fig. 1); in case (a), the same energies are measured by section *FG* and in case (c)—by *DE* respectively. As will be seen by confronting these sections, the maximum energy consumption on these rotational processes will be required in case (b) and the minimum consumption in case (c)*.

On account of the fact that with different temperatures the action of the given external field will be different, i. e. case (c) may with lower temperature pass into case (b), we must recognize our energy curves to be but a very approximate illustration of what takes place in reality. The energy curve will vary in approximately the same way in other crystals having a different orientation relatively to the field.

The above described idealized representation of the processes taking place during the cooling of a ferromagnetic polycrystal in a magnetic field involves as its result the fact that, when we select the magnitude of the external field during heat treatment, we therethrough are able to deform the magnetization curve and also to bring up to extreme values those magnetic parameters, which are of the greatest theoretical and practical interest.

3. The conclusions that can be drawn from the above scheme stand out with particularly bold relief, if the case of the heat treatment of transformer steel be considered. This steel had a sufficiently high Curie point ($\sim$730° C); with this steel magnetostriction at room temperature grows uniformly with field ([9]), and in this steel, in analogy with the behaviour of iron, we are apt to expect that this property will only become enhanced with increase of temperature ([10]); further the crystal anisotropy of this steel is sufficiently great for being able to differentiate clearly between the processes of inversion and rotation, beginning already from the region of average temperatures (300—400° and below). These are the reasons why we selected for our purpose silicon steel with 4% Si.

The specimens we took had the form of bands, $150 \times 50 \times 0.8$ mm^3 in size. They were first pickled in nitric acid for removing a layer of scale and were next annealed at 1000 °C during 3 hours in an atmosphere of running dry

---

* As has been shown experimentally by us([6]) and theoretically by Vonsovsky([7]), the theoretical positions of energy minima and maxima correspond to the minimum and maximum values of coercitive force in a Fe—Si monocrystal disc.

It has further been shown([8]) that annealing in a magnetic field changes the natural crystalline anisotropy H_c which appears to be the result of a change in the free energy of magnetic anisotropy in result of annealing.

hydrogen. This was done both for decarbonization and also to contribute, in so far as possible, to the completion of recrystallization processes, so that, with the subsequent short-time anneals at 800° C (required according to the conditions of the experiment), there should not be felt in any appreciable manner the effect upon magnetic properties of heat treatment as such. After that the specimens were placed in a quartz tube which was evacuated down to a high vacuum ($\sim 10^{-5}$ Hg). A furnace was next mounted on the tube and the specimen was heated up to $\sim$800° C; thereupon power was switched off and the specimen was allowed to cool down slowly with $H=0$. Curve *a* in Fig. 2 represents the magnetization curve of a specimen so treated. After this the specimen was annealed again under the same conditions, but only, on passing the Curie point, a magnetic field of the order of 1 Oersted (created by a solenoid mounted over the heating furnace) was switched in (with due allowance for demagnetizing effect). Curve *b* shows the magnetization obtained after such treatment. Lastly, the specimen was heated again and thereupon cooled, but this time in a field of the order of 85 Oersted. This gave magnetization curve *c*. Fig. 3 shows permeability in the mean fields of the same specimen.

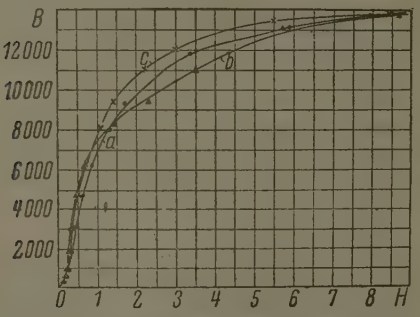


Fig. 2. Heat treatment: *a*—without field; *b*—in a weak field— 1 Oe, *c*—in a strong field—80 Oe.

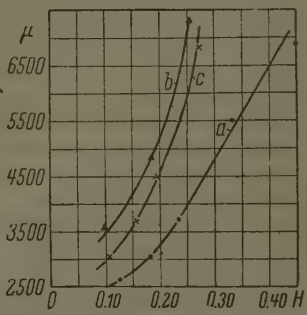


Fig. 3. Heat treatment: *a*—without field; *b*—in a weak field; *c*—strong field.

The many times repeated performance of the same operations with the same specimens reproduced every time the curves represented in Fig. 2 with very small scattering. A number of other specimens which underwent the same heat treatment gave magnetization curves similar to those represented in Fig. 2.

4. In comparing curves *a*, *b* and *c*, we find that in the sections of the magnetization curve caused by inversion processes these processes take place with the greatest ease in case (b) and run their course with the greatest difficulty in case (a); while in the section of the magnetization curve, where rotation processes play a predominant part, they occur with the greatest ease in case (c) and with the greatest difficulty in case (b). This agrees fully with the conclusions of the above given theoretical analysis. These results show that in order to obtain optimum magnetic properties, it is necessary to be very strict in selecting both the magnitude of the outer field and the rate of passage of various temperature fields. In particular, in order to obtain considerable initial permeability it is necessary to cool the ferromagnetic substance in a weak field, whereas in order to obtain the highest possible permeability the cooling has to be effected in a strong field. The highest effect in the sense of improving magnetic properties may appa-

rently be obtained in the case, if on cooling the substance the application of the magnetic field is varied so, that the spontaneous regions should all the time remain oriented in the directions needed.

Ural Physico-Technical Institute.
Sverdlovsk.

Received
10. II. 1939.

REFERENCES

[1] Pender a. Jones, Phys. Rev., 1, 259 (1913); Bozorth, Dillinger, Kelsall, Phys. Rev., 45, 742 (1934); O. Dahlu. T. Pawlek, ZS. f. Phys., 94, 504 (1935). [2] R. Bozorth a. J. Dillinger, Physics, 6, 276 (1935). [3] R. Bozorth, Phys. Rev., 46, 232 (1934); Physics, 6, 285 (1935). [4] R. Bozorth, Journ. appl. Phys., 8, 584 (1937). [5] R. Piety, Phys. Rev., 50, 1173 (1936). [6] J. S. Shur, C. R. Acad. Sci. URSS, XXI, 19 (1938); Журн. техн. физ., 8 (1938). [7] С. В. Вонсовский, Журн. техн. физ., 8 (1938). [8] R. I. Janus a. J. S. Shur, C. R. Acad.Sci. URSS, XX, 287 (1938). [9] A. Schulze, ZS. f. Phys., 50, 449 (1928). [10] H. Takaki, ibid., 105, 92 (1937).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

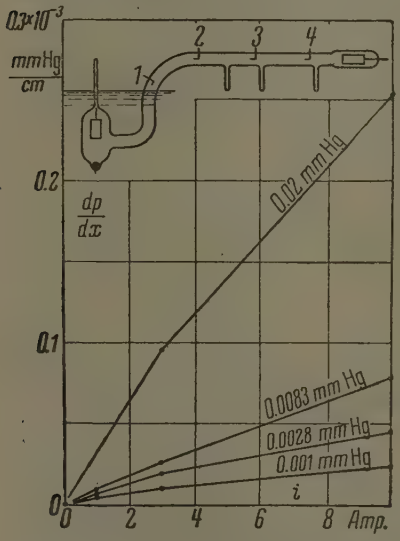
PHYSICS

PRESSURE GRADIENT IN THE POSITIVE COLUMN

By B. KLARFELD and I. POLETAEV

(Communicated by S. I. Wawilow, Member of the Academy, 28. III. 1939)

The charged particles of a discharge act in such a way on the molecules of the gas as to produce inhomogeneity in the pressure. This effect was experimentally investigated in inert gases by Hamburger (1) and Rüttenauer (2). A theoretical explanation was given by Langmuir (3), and later developed for the range of application of the diffusion discharge theory by Druyvesteyn (4). For the range of extremely low pressures, the only published material is that recently given by Kenty (5). In the investigation about to be described, the pressure changes in a mercury discharge were studied under various conditions. At the lowest pressure, experimental data are compared with calculated values.



Longitudinal pressure gradient in the positive column. Discharge tube diameter, 32 mm.

1. Method of Measuring Pressure Gradient. The cathode end of the discharge tube (Fig. 1) was immersed in a water bath, whose temperature could be regulated to give the desired mercury vapour pressure. This pressure was measured by the temperature, at which condensate appeared in the small side tubes which were also immersed in water («dew point»). A very sensitive method of detecting the appearance of condensate in the side tubes was by quickly heating the tube; vaporization, in this way, of minute quantities of mercury on the tube walls, instantly increased the intensity of illumination of the discharge. The condensation temperature could be determined to 0.5°C by this method. The vapour pressure corresponding to the temperature of condensation was taken to agree with the correction found by Mason (6), for mercury condensing on glass. At pressure of 0.001 mm and less,

measurements were carried out with the aid of an ionization gauge, calibrated using data on the saturated vapour pressure of Hg, allowance being made for thermal effusion.

2. Results of Measurements in a Uniform Positive Column. The axial pressure gradient measured by means of 2 side tubes is shown in the figure. For each series of measurements, the cathode temperature was so regulated that the arithmetical mean of the pressure at the 2 side tubes was constant. The spacing between the tubes had been chosen such as to give a difference in the two pressures not exceeding 40%. As seen from the figure, the gradient increases slower than proportionally to the current, and approximately proportionally to the vapour pressure. All the data refer to a discharge tube of 32 mm dia. The potential gradient, electron temperature and electron concentration could be measured by a number of cylindrical probes which were located at various points along the tube axis. The probe measurements showed, as might have been anticipated, the following: a) increase in potential gradient; b) increase in electron concentration; c) reduction in electron temperature, proceeding in the direction from cathode to anode. This emphasizes the necessity of taking into account the axial pressure gradient, when working with large discharge current*.

3. Comparison of Experimental Values of the Axial Pressure Gradient with Theory. In the case of such low pressures that practically all positive ions travel to the walls from their places of origin without collision with gas molecules, the axial pressure gradient can be calculated using Langmuir's conception of the nature of the phenomenon. The following simplifying assumptions will be made:

1) The positive column is considered as homogeneous across its section.

2) The translocation of matter along the column under the influence of electron momentum, as a result of motion of ions towards the cathode and due to pressure difference, are considered to be mutually independent. An expression for the pressure gradient is obtained by equating the sum of the 3 separate components of matter flow to zero, as it must be in the steady state.

The amount of substance transferred to the cathode by positive ions, Q_1, is:

$$Q_1 = -\frac{i_{px}}{e} m_p \quad (x \text{ increases toward the anode}) \qquad (1)$$

where i_{px} is the total positive ion current toward the cathode; e — the charge per ion; m_p — the mass of an ion; i_{px} is determined as follows:

$$i_{px} = eN_e\bar{v}_{px} = \frac{eE}{2m_p}\bar{\tau}eN_e, \qquad (2)$$

where N_e — number of ions (or electrons) in 1 cm length of column; E — axial potential gradient; τ — mean time taken by ions to reach the walls from their place of origin. $\bar{\tau}$ can be replaced by the equivalent quantity $\frac{1}{\alpha}$, where α is the number of ions generated per second per

* The error introduced by axial pressure gradient into the results published by one of us (Techn. Phys. of the USSR, 4, № 1, p. 44, 1937), does not amount to more than 20% at the lowest pressure used (0.00022 mm).

electron in the plasma; α can be substituted by its equivalent from the Tonks-Langmuir (7) theory for a low pressure cylindrical plasma:

$$\bar{\tau} = \frac{1}{\alpha} = \frac{a}{s_0}\left(\frac{m_p}{2kT_e}\right)^{\frac{1}{2}}. \qquad (3)$$

In this equation T_e is electron temperature, °K; a — tube radius; k — Boltzman constant; s_0 — dimensionless constant equal to 0.7722.

A more accurate analysis shows that in place of the average value $\bar{\tau}$, the root-mean-square should be used; however, the error in calculating i_{px} from this source is not large.* Thus, for Q_1 we obtain:

$$Q_1 = \frac{a}{s_0}\left(\frac{m_p}{8kT_e}\right)^{\frac{1}{2}} eN_eE. \qquad (4)$$

The amount of gas, Q_2, carried to the anode under the influence of electron impacts can be determined by the force exerted on the gas molecules and the resistance of the tube to the flow of gas. The latter is at low pressures expressed by Knudsen's formula. The positive ions and the electrons, owing to their practically equal densities in the plasma, acquire under the action of the longitudinal field equal, but oppositely directed, total momenta. Since the ions rarely collide with gas molecules, the latter acquire anode-ward momentum only from the electrons. The total magnitude of this momentum, M, per cm length of tube is

$$M = eEN_e\beta, \qquad (5)$$

where β is a factor giving the fraction of the electron momentum which is passed on to the gas. The fraction $(1-\beta)$ gives the part of the electron momentum which is passed to the positive ions in the volume. This part cannot cause a flow of gas to the anode. The electron momentum lost at the walls by electrons recombining there has been neglected. It is not difficult to show that this part amounts to about 1% of M, in mercury vapour. Thus, Q_2 is equal to

$$Q_2 = \frac{4}{3}(2\pi\rho_1)^{\frac{1}{2}} a^3 \frac{eEN_e\beta}{\pi a^2}, \qquad (6)$$

where $\rho_1 = \frac{m_p}{kT_g}$ is the density of gas at 1 bar pressure.

---

* It is of interest to note that on applying the Langevin mobility equation to the electrons:

$$i_{ex} = 0.75\,\frac{e\lambda_e}{m_e}\left(\frac{\pi m_e}{8kT_e}\right)^{\frac{1}{2}} eN_eE$$

and substituting in (2) and (3), we get

$$\frac{i_{ex}}{i_p} = 0.75 s_0 \sqrt{\pi}\,\frac{\lambda_e}{a}\left(\frac{m_p}{m_e}\right)^{\frac{1}{2}} \approx \frac{\lambda_e}{a}\left(\frac{m_p}{m_e}\right)^{\frac{1}{2}}.$$

At low pressures ($p < 0.001$ mm) and with $a =$ several cm, the ratio $\frac{i_{ex}}{i_{px}}$ in mercury vapour will be more than an order of magnitude greater than the ordinarily accepted value of $\left(\frac{m_p}{m_e}\right)^{\frac{1}{2}}$.

The amount of gas, Q_3, transferred as a result of the pressure gradient, is equal to

$$Q_3 = -\frac{4}{3}(2\pi p_1)^{\frac{1}{2}} a^3 \frac{dp}{dx}. \qquad (7)$$

Equating $(Q_1+Q_2+Q_3)$ to zero,

$$\frac{dp}{dx} = \frac{eEN_e}{\pi a^2}\left(\beta - \frac{3\sqrt{\pi}}{16 s_0}\sqrt{\frac{T_g}{T_e}}\right) = 1.19\cdot 10^{-15}\frac{EN_e}{\pi a^2}\left(\beta - 0.43\sqrt{\frac{T_g}{T_e}}\right). \qquad (8)$$

There T_g is temperature of the gas which at low pressure is practically equal to the wall temperature. The numerical coefficient is based on values of E in volts/cm, and $\frac{dp}{dx}$ in mm Hg per cm.

The quantity β is determined by the expression

$$\beta = \frac{N_0 q_0}{N_0 q_0 + N_e q_+}.$$

N_0 is number of molecules per cm length of tube; q_0 — molecular cross-section for electron momentum transfer; q_+ — positive ion cross-section.

The value of $(N_0 q_0 + N_e q_+)$ can be obtained from equation (7) of Killian's paper ([8]) and q_+ from the formula derived by Gvosdover ([9]). We then get the following expression for β:

$$\beta = 1 - 2.02\cdot 10^3 \frac{i}{a^2 E T_e^{\frac{2}{3}}} \ln \frac{9}{4}\frac{k^2 T_e^2}{e^4 n_e^{\frac{2}{3}}}. \qquad (9)$$

A comparison of values of $\frac{dp}{dx}$, obtained experimentally and calculated by use of equations (8) and (9), is given in the table. The values given in the table are the averages over the considered section of the tube.

Average pressure, mm Hg	i, amps	N_e, 1/cm	T_e, °K	E, V/cm	dp/dx, mm Hg/cm	
					Experimental	Calculated
0.0004	0.3	$0.52\cdot 10^{11}$	33.400	0.10	$0.085\cdot 10^{-5}$	$0.067\cdot 10^{-5}$
	1.0	1.84	30.400	0.11	0.16	0.21
	3.0	5.04	42.400	0.10	0.22	0.51
	10.0	17.9	44.500	0.06	0.36	0.87
0.001*	1.0	2.16	28.100	0.17	0.24	0.39
	3.0	7.85	28.100	0.165	1.0	1.38
	10.0	28.0	28.500	0.14	2.4	3.28

It is clear from these data that the experimental values of dp/dx agree with those calculated at small currents, and begin to diverge from them at large currents. It is still hard to say whether this lack of agreement is due to a somewhat diffuse reflection of electrons from the ion

* At a pressure of 0.001 mm Hg collisions of positive ions with molecules, en route to the walls, are capable of causing an error in calculation of not more than 20%.

layer on walls ([3], [5], [9])*, the diffuseness increasing with current; to inhomogeneity of the discharge across the tube section; or, finally, to some other effect not considered in the calculation. In any case, it is essential to point out that at small discharge currents the experimental data are well explained under the assumption of specular reflection of electron from the spatial charge layer at the walls.

It must further be added that at the lowest pressure ($p \approx 5 \times 10^{-4}$ mm Hg), a jump in potential which could be measured with probes 1 and 2, occurred at the bend in the tube (point A, figure). The magnitude of this jump rises rapidly with the discharge current from a very small value up to several volts, and has a tendency to distort considerably all the discharge characteristics. For example, the potential gradient measured by probes 2 and 4 is reduced 2.5 times on increasing the current from 3 to 10 amps, and reaches the values of 0.04 volt/cm. The probe current curves are distorted in a characteristic manner. Under these conditions, the positive column loses all similitude to homogeneity along its length and, naturally, the application of the above calculations becomes impossible.

All-Union Electrotechnical Institute.
Moscow.

Received
28. III. 1939.

REFERENCES

[1] L. Hamburger, Proc. Amst. Soc., **20**, 1043 (1917). [2] A. Rüttenauer, ZS. Physik, **10**, 269 (1923). [3] I. Langmuir, Jl. Franklin Inst., **196**, 751 (1923). [4] M. Druyvesteyn, Physica, **2**, 225 (1935). [5] C. Kenty, Jl. Appl. Phys., **9**, 765 (1938). [6] R. Mason, Jl. Appl. Phys., **9**, 131 (1938). [7] L. Tonks a. I. Langmuir, Phys. Rev., **34**, 876 (1929). [8] T. Killian, Phys. Rev., **35**, 1238 (1930). [9] S. Gvosdover, Sow. Phys., **12**, 164 (1937).

* If the range of application of (8) is to be made more general by consideration of the diffuseness of reflection of electrons from the ions sheath, it is possible to use the coefficient ε proposed by Langmuir. This coefficient expresses the fraction of the momentum of the electrons lost in the body of the gas [ε is supposed equal to unity, in deriving (8)]. Then in place of β, the product $\beta\varepsilon$ must be used in (8).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

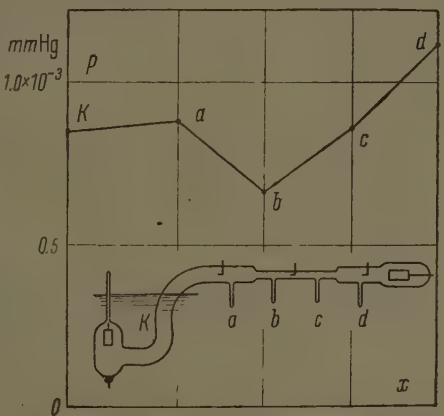
PHYSICS

GAS RAREFACTION AT CONSTRICTIONS IN THE POSITIVE COLUMN

By B. KLARFELD and I. POLETAEV

(Communicated by S. I. Wawilow, Member of the Academy, 28.III.1939)

If the positive column of a discharge contains a constriction, a sudden potential drop occurs at its cathode end. Upon analyzing phenomena in the positive column, Tonks (1) came to the conclusion that such a potential drop would be able to cause rarefaction of the gas at the constriction. Rarefaction is brought about by the transfer of momentum to the gas molecules, in opposite directions, by the ions and electrons which are accelerated in the potential drop. It must be added that the effect of the positive ions here is strengthened by the intense ionization in the constriction, brought about by electrons which have just been accelerated in the intensified field. Thus, the fraction of the discharge current carried by positive ions is increased in this region.



Distribution of vapour pressure near a constriction in a discharge tube. Current—5 amps, tube diam.—40 mm, constriction diam.—14 mm.

The experiments to be described demonstrate the existence of such a rarefaction in a mercury discharge at low pressure. The discharge tube employed is shown in figure. The projecting branches were used for determining the vapour pressure from the «dew point» (2). At currents of about 5 amps and more, and at vapour pressures of the order of 0.001 mm, the following phenomena pointed to the presence of rarefaction.

1) The intensity of illumination of the discharge was less in the constricted than in the normal sections of the tube, in spite of the increased current density. This is the opposite to what happens in high pressure discharges.

2) Direct pressure measurements by means of the dew point showed a decrease in pressure as compared to that on the cathode side, shown in figure.

3) Probe current characteristics taken in the center of the constriction show a distortion typical of a discharge, where the electron current has a highly directional character, while a probe in the broad cathode section gives an ideal linear characteristic on a semi-log plot.

On increasing the discharge current, the walls at the cathode end of the constriction begin to heat intensely, the illumination from the discharge becomes very weak, and at a certain value of current the discharge suddenly extinguishes, apparently as a result of extreme rarefaction. Extinction of the discharge is apparently facilitated further by the rarefaction occurring at the axis of low pressure discharge. At current densities of the order of 1 amp/cm^2, several per cent of the vapour atoms are in the ionized state, and since the outward velocity of the ions under the influence of the radial field is several times greater than the thermal velocity with which they return to the central volume after recombination at the walls, a considerable rarefaction at the axis may take place. The experimental determination of such an effect would be very complicated, because of the difficulty of measuring the gas density at the axis.

The described phenomena taking place at the tube constriction and similar ones occurring at its bends, must be borne in mind when working with large discharge currents. It is quite possible that a number of the interesting phenomena observed by F. Mohler ([3]) in caesium vapour, can be explained by a lowering of pressure in the constriction in which the measurements were carried out.

All-Union Electrotechnical Institute.
Moscow.

Received
28. III. 1939.

REFERENCES

[1] L. Tonks, Trans. El. Chem. Soc., 72, 1 (1938). [2] B. Klarfeld a. I. Poletaev, C. R. Acad. Sci. URSS, XXIII, № 5 (1939). [3] F. Mohler, B. S. J. R., 21, 697—873 (1938).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

CHEMISTRY

ANAEROBIC BREAKDOWN OF PHOSPHOGLUCONIC ACID IN THE PRESENCE OF ACETALDEHYDE

By A. P. BARKHASH

(Communicated by A. N. Bach, Member of the Academy, 19.III.1939)

In a previous communication(1) evidence has been produced that the oxidative breakdown of phosphogluconic acid can be effected under anaerobic conditions in the presence of suitable hydrogen acceptors. It is of particular interest that this breakdown can take place in the presence of acetaldehyde. Considering that no proof has been previously given that the effect of added acetaldehyde depends upon its rôle as hydrogen acceptor in the dehydrogenation of phosphogluconic acid, we have submitted this reaction to a more detailed reinvestigation.

M e t h o d s. Yeast maceration juice contains considerable amounts of performed ethyl alcohol causing difficulties in the estimation of alcohol formation during the experiment. Therefore the juice used in this work was freed of the bulk of ethyl alcohol by preliminary evaporation to syrupy consistence under reduced pressure (about 1 mm) and a temperature of not more than 35°. The capacity of the juice to effect the breakdown of phosphogluconic acid is not substantially impaired by this treatment, whereas the dismutative action on the added acetaldehyde is considerably decreased, thus also eliminating a source of error in the final determinations.

For use the condensed syrup was diluted with water to the original volume of yeast juice. The reaction course was followed by manometric measurement in Warburg respirometers and with chemical methods. Anaerobic conditions were ensured by passing adequately purified N_2 through the Warburg vessels at 0° during 10 min. at a standard rate of 100 ml per minute.

This involves the unavoidable loss of a certain amount of acetaldehyde, but this loss can be accounted for and does not surpass 20% under our standard conditions (cf. Dixon and Lutwak-Mann(2)]. A slight CO_2-retention taking place in spite of the weakly acid reaction of the medium was accounted for by acidification of the experimental sample at the end of the experiment, along with the control sample. The chemical determinations were performed upon filtrates from the samples after deproteinization with metaphosphoric acid.

The aldehyde was estimated after distillation in the Lieb-Zacherl apparatus by the usual bisulphite titration method. The sum of volatile bichromate reducing substances (aldehyde+alcohol) was determined after Widmark: 1.0 ml of the protein-free filtrate from the reaction mixture, neutralized to phenolphthalein and containing about 2.5—5.0 μM alcohol along with approximately the same amount of acetaldehyde, was evaporated during 3 h. at 65—70°, the receiver flask containing 5.0 ml of *n*/50 bichromate-sulphuric acid mixture. From the data of the two analyses the alcohol content was deduced. In this calculation we were compelled to use an empirically established computation coefficient (1.29) for acetaldehyde, because oxidation by the bichromate mixture of aldehyde, contrary to the oxidation of alcohol, does not proceed stoichiometrically, as usually thought [for instance, cf. Warburg and Chirstian(3)], but goes somewhat beyond the theoretical limit. The sampling of all fluids containing acetaldehyde was carried out at 0°.

Table 1

Decrease of Aldehyde and Formation of Alcohol and CO_2 in N_2 Atmosphere (0.5 ml maceration juice, 20 µM phosphogluconate, 20 µM acetaldehyde; 28°; duration of expts 2½—3 h. From all data the corresponding values of control samples have been substracted)

Datum	+Alcohol µM	—Aldehyde µM	+ CO_2 µM
28.I	11.3	7.0	6.5
20.II	14.7	8.2	6.2
22.II	13.0	6.2	8.3

The experiments presented in Table 1 show that the anaerobic breakdown of phosphogluconic acid in the presence of acetaldehyde is actually associated with disappearance of aldehyde and formation of alcohol. The formation of 1 mol CO_2 is accompanied by the disappearance of about 1 mol acetaldehyde and the formation of about 2 mols alcohol.

On the basis of these results and the comparable previous data([1]) concerning the similar action of other acceptors, and taking into account the statements available in literature as to the primary products of oxidative breakdown of phosphogluconate ([4],[5]), we can assume that the oxidation of phosphogluconate by aldehyde involves dehydrogenation of the carbohydrate molecule followed by decarboxylation and resulting in the formation of the corresponding pentose derivative.

Substantial support for this assumption has been obtained by examining the reaction mixtures at the end of the experiment by the Bial test ([6]) for pentoses. This test invariably gave a strongly positive result.

The formation of alcohol observed in the experiments with acetaldehyde gives unequivocal evidence that the aldehyde actually functions as a hydrogen acceptor under these experimental conditions and undergoes hydrogenation to alcohol, as in alcoholic fermentation. However, as stated above, the amount of alcohol formed is not only equivalent, but even surpasses the disappearing quantity of aldehyde. It is evident that the alcohol formed in excess must arise from some transformation product of the phosphogluconate itself. It may be assumed that the primary process of dehydrogenation and decarboxylation of phosphogluconate is followed by a process of breakdown that does not require the participation of a foreign acceptor and proceeds according to the type of usual fermentation processes [cf. Lipmann([7])]. This is the process, owing to which alcohol is formed in our experiments in excess of the amount that can arise by way of reduction of the added aldehyde. The possibility of phosphopentose fermentation has been experimentally demonstrated by Dickens([8]).

If this interpretation is correct, it should be expected that an amount of alcohol similar to that which is formed in our experiments, in excess of the equivalent of disappearing aldehyde, will likewise be formed when phosphogluconate is broken down in the presence of other acceptors. In particular, this should be true for the breakdown under aerobic conditions when molecular oxygen acts as the hydrogen acceptor in the primary dehydrogenation. As seen from the data presented in Table 2, this is actually the case: the aerobic breakdown of phosphogluconate is associated with the formation of certain amount of alcohol, fairly close to the excess amount of alcohol formed in the experiments with added aldehyde.

Table 2

Alcohol Formation in Air (0.5 ml maceration juice, 20 μM phosphogluconate; 28°; duration of expts 2—2½ hs. Controls substracted from all data)

Datum	+ Alcohol μ M	$-O_2$ μ atomes	$+CO_2$ μ M
1.II	5.0	6.4	9.5
26.II	6.4	5.4	8.9

It was mentioned above that the reaction here described—the formation of CO_2 from phosphogluconate with the simultaneous formation of alcohol at the expense of aldehyde hydrogenation—might be considered from the formal standpoint as a peculiar type of alcoholic fermentation. Are we entitled to attribute any importance to this reaction as one of the routes of carbohydrate breakdown in fermentation? Is there sufficient evidence for assuming the possibility of carbohydrate fermentation by gradual shortening of the carbon chain rather than by cleavage of the carbohydrate molecule into two halves? No definitive answer to these questions can be given as yet; the available data in support of a positive answer are few.

As opposed to the genuine fermentations, the output of CO_2 is not associated in our case with the continuous new formation of the oxidant; on the contrary, it involves the expenditure of foreign oxidants. This is why the process does not exhibit the cyclic autocatalytic linkage of intermediate phases that is characteristic of fermentation.

The author's thanks are due to Prof. V. A. Engelhardt for valuable advice in the course of this investigation.

Institute of Biochemistry.
Academy of Sciences of the USSR.

Received
19. III. 1939.

REFERENCES

[1] В. А. Энгельгардт и А. П. Бархаш, Биохимия, **3**, 500 (1938). [2] M. Dixon a. C. Lutwak-Mann, Biochem. Jl., **31**, 1347 (1937). [3] O. Warburg a. W. Christian, Biochem. Z., **287**, 291 (1937). [4] O. Warburg a. a. W. Christian, ibid., **292**, 287 (1937). [5] F. Dickens, Biochem. Jl., **32**, 1626 (1938). [6] Tollens-Elsner, Kurz. Lehrbuch d. Kohlenhydrate (1935). [7] F. Lipmann, Nature, **138**, 588 (1936). [8] F. Dickens, Bioch. Jl., **32**, 1645 (1938).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

MINERALOGY

SOME ANOMALIES IN THE PROPERTIES OF CASSITERITE

By J. D. GOTMAN

(Communicated by A. E. Fersman, Member of the Academy, 26.III.1939)

In course of the systematic investigation of the optical properties of cassiterites obtained from a number of tin deposits of the USSR, certain anomalies have been traced which deserve further investigation. Such anomalies have been known for some time; they were recorded by Rosenbusch(1), who pointed to their rarity and small value*; they were also mentioned by Winchell and Winchell (2), as well as by Lodochnikoff (3). The latter, while considering the anomalies of cassiterite to be very rare, gives as the maximum magnitude of the angle $2V=5°$.

We have determined the indexes of refraction** N_0 for different cassiterites by the lens method described by Wedenejeva, Grum-Grzimaylo and Wolkoff (4). We used alloys of piperine with the iodides of As and Sb. In this method two small doses of the alloy are applied on to one object glass. The lens is placed on one of these doses, while the powder of the mineral is mixed with the other one, the whole mixture being covered by a cover glass. The object glass with both doses of the alloy is then heated on an electrical range, whereupon, after melting and subsequent cooling, the index of refraction of the alloy is determined by means of an interferometer with the help of the lens. At the same time, for the dose of the alloy with the grains of the mineral fused into it, the indexes of refraction of the alloy and the mineral are made to be equal in monochromatic light. Since the heating of both doses of the alloy takes place simultaneously, the indexes of refraction of both these doses after melting and subsequent cooling are the same, and the authors believe that in this way the precision of the determination is brought up to 0.003. Owing to the fact that cassiterite possesses but a single axis and as there are fluctuations along N_0 for each cut of this mineral, it is possible to make use of any fragment of a cassiterite grain for determination.

In measuring the indexes of refraction N_0 for cassiterites, it was found that cassiterites from different deposits and in different grains gave a different value of N_0, the deviations falling out of the range of the precision mentioned above, namely:

* In the older, third, edition of his book in 1892 Rosenbusch writes that he frequently observed a cross being broken up into hyperboles while rotating the thin-section.

** The indexes of refraction have been determined by the author, who pays his tribute of acknowledgment to the physicist A. T. Kolotushkin who gave him the necessary consultations.

		N_o for the light		N_a
Cassiterite from	Zun-Undursk deposit . .	1.999	and	2.004
» »	Urunkhaisky » . .	2.001	»	2.013
» »	Sokhondinsky » . .	1.989	»	1.999

The above-mentioned deviations in the values of the indexes of refraction cannot be accounted for by the insufficient exactitude of the methods because we carried out parallel determinations for a number of cassiterites which coincided well with one another. Accordingly, there is need to seek for an explanation of these deviations in the anomalous optical properties of the cassiterites, assuming that the greater values from among the indexes measured refer to N_m or $N_{m'}$.

A study of cassiterites in thin-sections gave additional support to the existence of such anomalies. From among the cassiterites investigated obtained from 14 tin deposits of the USSR, cassiterites from 6 deposits were found to be biaxial. In the zonal cassiterite from the Takfonsky deposit, in one of the non-coloured zones, a very inconsiderable divergence of the hyperboles of the cross was observed; in another non-coloured zone and in a zone coloured reddish-brown of the same cassiterite the deviation of the hyperboles of the cross was greater. Zonal cassiterites from the Cherdoyak, Iultinsky and Shumilovsky deposits are manifestly biaxial. Still greater values of angle $2V$ have been observed in the cassiterites of the Urunkhaisky and Sokhondinsky deposits, for which we found indexes of refraction greatly differing in their magnitude.

The angle $2V$ in the cassiterites of the Urunkhaisky, Sokhondinsky and Shumilovsky deposits was so considerable that it was not difficult to measure it. However, owing to the great double-refraction of cassiterite and also on account of considerable dispersion, we succeeded in determining the positions of its optical axes only in monochromatic light. Beside this, the great difference in the indexes of refraction of the mineral and segments of the Fedorov stage required the introduction of considerable corrections. Therefore, the below given data of $2V$ angles measured on the Fedorov stage in yellow light are certainly somewhat less exact than the results obtained from ordinary rock-forming minerals.

Deposit	Values $2V$ measured on the Fedorov stage in yellow light
Cassiterite of the Sokhondinsky deposit. Non-coloured part of the grain	38°
The same cassiterite. Red-coloured zone of the same grain	21°
Cassiterite of the Urunkhaisky deposit. Non-coloured part of the grain	25°
Cassiterite of the Shumilovsky deposit. Non-coloured part of the grain	0°
The same cassiterite. Red-coloured zone of the same grain	9°
The same cassiterite. The next non-coloured zone of the same grain	0°

In addition to this, it is necessary to emphasize that all biaxial cassiterites are more or less zonal and have a spotted colouring. Coupled with

grains and zones that are anomalously biaxial, there are to be found grains and zones for which it is impossible to observe any divergence in hyperboles. The sign of all anomalously biaxial cassiterites remains positive.

For the cassiterite from the Zun-Undursky deposit we failed to detect any such anomalies, though, as can be seen from the above-given data, fluctuations in the magnitude of the index of refraction are here also observed. This can possibly be explained by the insufficiency of the material at our disposal. In our thin-sections we found but a few suitable cuts, while in other sections with such comparatively small $2V$ angles, it appears exceedingly difficult to seek for anomalies with the help of the Fedorov stage, even in case when monochromatic light is resorted to.

The evidence given above affords us the possibility of drawing the following conclusions:

1. Anomalies in the optical properties of cassiterite occur much more frequently than is generally believed to be the case. The angle of the optical axes in this mineral may also reach much higher values than stated in the literature.

2. Anomalies of properties, i. e. changes in the crystalline lattice of cassiterite, are observable in cassiterites of zonal structure and are apparently connected with the zonal growth of crystals, stronger anomalies being observed both in the coloured and in the non-coloured zones of the crystal.

3. All the facts stated above lend indirect evidential support to the opinion advanced by the author formerly([5]) to the effect that the colouring of cassiterite is not caused by the colouring properties of admixtures, but by those properties in the crystalline lattice of cassiterite which are caused by this lattice being pervaded by admixtures. From this point of view, cassiterite may contain impurities and remain non-coloured with changed crystalline lattice and anomalous optical properties.

Institute of Geological Sciences.
Academy of Sciences of the USSR.
Moscow.

Received
27. III. 1939.

REFERENCES

[1] H. Rosenbusch., Mikroskop. Physiographie d. petrogr. wichtigen Mineralien, I, zweite Hälfte, fünfte erweiterte Auflage (1927). [2] N. H. Winchell a. A. N. Winchell, Elements of Optical Mineralogy, P. II (1927). [3] В. Н. Лодочников, Главнейшие породообразующие минералы, 2 изд. (1938). [4] N. Wedenejewa, S. Grum-Grzimaylo u. A. Wolkoff, Acta Physicochimica URSS, 5, № 3. [5] Я. Д. Готман, Бюлл. Моск. о-ва испыт. прир., отд. геологии, XVI (2) (1938).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

GENETICS

SECOND GENOM OF *TRITICUM TIMOPHEEVI* ZHUK.

By V. V. SVETOZAROVA

(Communicated by N. I. Vavilov, Member of the Academy, 21.III.1939)

For obtaining immune varieties of wheat by means of synthetic breeding, *Triticum Timopheevi* Zhuk. is of great interest (1). It is, therefore, very important to know the phylogenetic relation of this species, which stands somewhat apart to other species of wheat.

Timopheevi wheat was first found in 1922 in eastern Georgia and was classified as a variety of wild emmer, *T. dicoccum dicoccoides* (Körn.) Schulz. with which it has several morphological characters in common (2). Subsequently, it was noted that *Tr. Timopheevi* had a number of characters of cultivated wheat (3) and that it occupied an intermediate position, as a Transcaucasian *Tr. dicoccoides*, between Syrio-Palestinian *dicoccoides* and *T. dicoccum* (4). Finally, in 1928, Zhukovsky gave it the rank of an independent species, *T. Timopheevi* Zhuk. (5). This species is localized primarily in western Georgia (6), only one specimen having been found in eastern Georgia (2).

The first cyto-genetic investigations showed that the new species occupies a unique position among 28-chromosome wheats (7–9). The genomic composition of *T. Timopheevi* was considered by some investigators to be *AAgg* (9) and by others to be *AA* ββ (10). Thus, it was established that *T. Timopheevi* occupies a special position among wheats, but its phylogenetic relations still remained somewhat unclear.

We undertook the task of studying this problem, and for this purpose we crossed *T. Timopheevi* with all the principal species and varieties of wheat and subjected the hybrids thus obtained to morphological and cytological analysis. The hybrids *T. Timopheevi* × *T. dicoccoides* had not previously been studied (11). Of particular interest was the analysis of the hybrids *T. Timopheevi* × *T. dicoccoides* Körn. subsp. *armeniacum* Jakubz., the latter having recently been given by E. N. Makushina the rank of an independent species, *T. armeniacum* (Jakubz.) Makusch. (12). Such special interest arose from the fact that both parental species are Transcaucasian endemics and are comparatively similar in morphological characters.

The present paper gives the results of a study of these hybrids. The two varieties of *T. Timopheevi* (*typicum* and *viticulosum*) were crossed with the following varieties of *T. dicoccoides* Körn.: *spontaneo-nigrum* Flaksb., *jordanicum* Vav. (proles *horanum* Vav.), *arabicum* Jakubz. (proles *judaicum* Vav.), *spontaneum* Jakubz. (proles *incertum* Jakubz.). *T. armeniacum* (Jakubz.) Makusch. was represented by two varieties: *Tumaniani* Jakubz. and *nachitschevanicum* Jakubz.

Table 1

Crossability of Parental Forms (in %)

No. of flowers pollinated	Successful crosses (in %)	Combination
116	19.8	*T. Timopheevi* × *T. armeniacum* var. *Tumaniani*
172	25.0	*T. Timopheevi* × *T. armeniacum* var. *nachitschevanicum*
170	10.6	*T. Timopheevi* × *T. dicoccoides* var. *arabicum*
213	11.7	*T. Timopheevi* × *T. dicoccoides* var. *spontaneo-nigrum*
532	15.9	*T. Timopheevi* × *T. dicoccoides* var. *jordanicum*
366	18.8	*T. Timopheevi* × *T. dicoccoides* var. *spontaneum*

Table 2

Fertility of F_1 (open pollination)

No. of spikes	Total No of kernels set	Aver. No. of kernels per spike
153	14	0.09
819	94	0.11
210	17	0.08
185	—	—
415	33	0.07
330	6	0.02

The results of the crosses, made in 1936 in Pushkin (near Leningrad) are given in Table 1. Since there was no perceptible difference between direct and reciprocal crosses in the given combinations, the data for both have been combined. On the basis of these data we may conclude that *T. Timopheevi* hybridizes somewhat more readily with *T. armeniacum* (crosses, on the average, 22.9% successful) than with *T. dicoccoides* (crosses, on the average, 13.7% successful). *T. armeniacum*, however, does not constitute an exception in this respect. Other 28-chromosome species of wheat crossed no less readily with *T. Timopheevi*.

Morphologically the F_1 hybrids, as regards a number of characters, were intermediate, but the general type of *Tr. dicoccoides* or *Tr. armeniacum* was dominant. The anthers did not dehisce; the spikes were left uncovered at time of flowering. From bagged hybrid spikes we did not obtain a single kernel, and even from open pollination the percentage of kernels set was very insignificant (Table 2).

As was to be anticipated, meiosis in the F_1 hybrids *T. Timopheevi* × *T. dicoccoides* proved to be irregular. Thus, at meiosis in *T. Timopheevi* × *T. dicoccoides* var. *jordanicum* the number of bivalents ranged from 7 to 10 (most frequently 10). Of these from 2 to 5 had two or more chiasmata and from 5 to 8 (usually 6) had one chiasma. There were occasional pollen mother-cells (PMC's) with multivalents (Fig. 1). As a result of irregular meiosis micronuclei were present in the tetrad cells. The same irregularities at meiosis were observed in the combination *T. Timopheevi* × *T. dicoccoides* var. *spontaneum*. This is in accord with published data on hybrids between *T. Timopheevi* and 28-chromosome wheats ([9–11]).

Contrary to expectations, however, meiosis in the hybrids *T. Timopheevi* × *T. armeniacum* var. *Tumaniani* proved to be markedly different from that in the preceding combinations. Not a single time did we come across univalents. In Table 3 are given data for only 25 PMC's (from 4 plants at various times); actually we investigated over 100. As is seen from Table 3, chromosome conjugation was regular. The number of bivalents with two or more chiasmata ranged from 8 to 14 (Figs. 2, 3). In all they constituted 85.7% of the total number of chromosomes. Bivalents with but one chiasma made up

only 6.3% (Fig. 4) and quadrivalents 8% of the total number of chromosomes. Of all the PMC's 36% had only bivalents with two or more chiasmata and 12% had 12 bivalents with two or more chiasmata plus 1 quadrivalent, while the remaining 52% had from 1 to 3 bivalents with but one chiasma. At first metaphase in almost half (44%) of the cases we found quadrivalents (in 36% of the PMC's there was one quadrivalent and in 12% two), and they often took the form of closed rings (Fig. 5). At the first anaphase there were no lagging chromosomes. In very rare cases there were observed chromatin bridges. At the first anaphase the chromosome distribution was usually 14—14, only in very rare cases 15—13. It was not possible to study the second division in all details. From the plates studied, and also judging by the fact

Fig. 1. *T. Timopheevi* × *T. dicoccoides* var. *jordanicum*. I Metaphase. 7_{II} (5_{0II} with one chiasma, 2_{II} with two or more chiasmata). 14_{I}.

Fig. 2. *T. Timopheevi* × *T. armeniacum* var. *Tumaniani*. Diakinesis. 14_{II}.

Fig. 3. *T. Timopheevi* × *T. armeniacum* var. *Tumaniani*. I Metaphase. 14_{II} with two or more chiasmata.

Fig. 4. *T. Timopheevi* × *T. armeniacum* var. *Tumaniani*. I Metaphase. 10_{II} with two or more chiasmata; 2_{II} with one chiasma; 1_{IV}.

Fig. 5. *T. Timopheevi* × *T. armeniacum* var. *Tumaniani*. I Metaphase. 12_{II} with two or more chiasmata; 1_{IV} in the form of a ring.

that perfectly regular tetrads (without micronuclei or microcytes) were formed, it may be concluded that this division proceeds normally. However, a morphological analysis of the pollen, by means of aceto-carmine preparations, showed that only occasional pollen grains stained normally; most of the pollen was entirely deformed.

Meiosis in the hybrids *T. Timopheevi* × *T. armeniacum* var. *nachitschevanicum* was practically the same, the only difference being that PMC's with two quadrivalents occurred with greater frequency.

There are only very few papers devoted to the problem of pollen sterility in hybrids with regular chromosome conjugation and regular distribution of the chromosome to the poles. Such sterility may be ascribed to the following causes: 1) impossibility of the substitution, in the gametes, of the chromosomes of one parent for those of the other, due to the great divergence of the parental forms as a result of gene mutations or structural chromosome changes [cf. ([13])]; 2) effect of complimentary genes in a new genotype ([14]).

In our case it may be presumed that the inviability of the gametes is

Table 3

Conjugation of Chromosomes at First Metaphase
F_1 *Tr. Timopheevi*×*Tr. armeniacum*

No. of PMC's	No. of bivalents with one chiasma	No. of bivalent with two or more chiasmata	No. of quadrivalents
9	—	14	—
3	—	12	—
2	1	11	1
2	1	9	2
2	1	13	—
2	2	12	—
2	2	10	1
1	2	8	2
1	3	11	—
1	3	9	1

the result of the fact that the chromosomes of one species cannot be substituted for those of the other. In order to settle definitely the question as to the absence of genes of sterility, it is necessary to obtain an amphidiploid.

The presence of quadrivalents at meiosis points to the fact that in the evolution of *T. Timopheevi* and *T. armeniacum* reciprocal translocations between non-homologous chromosomes had definite significance. Presumably, it was precisely such translocations which laid the basis for the further divergence of these species. Translocations may have created such genetic conditions that, despite the relatively close affinity of these species, sterility results from their hybridization.

As we have already mentioned, two formulae have been given for the genomic constitution of *T. Timopheevi*: *AAgg* ([9]) and *AA* ββ ([10]). The latter formula presumes that the second genom ββ is only a modification of the genom *BB* of durum wheat. The grounds for this lie in the presence at meiosis in the hybrids between *T. Timopheevi* and *T. durum* or *T. persicum* of 10—12 bivalents, of which only two or three have two chiasmata.

Such a conclusion seems to us incorrect for the following reasons. There are data which show that autosyndesis occurs in *Triticum* ([15—17]). However, this does not give any grounds for calling the three wheat genoms modifications of one and the same genom; it merely points in general to the polyploid nature of wheat. The conjugation of some of the chromosomes of the second genom of *T. Timopheevi* with the chromosomes of *T. durum* and *T. persicum* we consider possible to regard as a phenomenon of approximately the same kind, i. e., indicating merely a distant phyletic relation between these two genoms, but nothing more.

We find additional grounds for considering that *T. Timopheevi* has a genom specific to it, if we compare meiosis in the hybrids *T. Timopheevi*×28-chromosome wheats with that in the hybrids *T. armeniacum* ×28-chromosome wheats ([12]). At meiosis in these hybrids we find approximately the same irregularities and the same percentage of sterile pollen. At the same time, meiosis in the hybrids *T. Timopheevi*×*T. armeniacum* proceeds, as we have seen, without noticeable irregularities. If, moreover, we recall that the two species are morphologically somewhat similar and are both endemics of Transcaucasia ([18]), we cannot but conclude, on the basis of our present knowledge as to the causes of conjugation, that *T. Timopheevi* and *T. armeniacum* have homologous genoms. Their formula will be *AAgg*. This conclusion as to the affinity of *T. armeniacum* and *T. Timopheevi* should, of course,

be understood only historically. Despite having homologous genoms, these species differ considerably in morphological and economico-biological characters. *T. armeniacum* is an endemic of mountainous and semi-desert regions in Armenia and Nakhichevan with xerophytic vegetation, while *T. Timopheevi* is an endemic of a narrow strip of territory in western Georgia having a typical mesophytic vegetation.

Whether or not the second genom of *T. armeniacum* will, upon further investigation, be regarded as a genom fully homologous to the second genom (genom *g*) of *T. Timopheevi*, or only a modification of the latter, it seems to us that the group of wheats standing apart from the others and represented heretofore by the single species, *T. Timopheevi* Zhuk. ([9]), should be supplemented by the new species, *T. armeniacum* (Jakubz.) Makusch. The differentiation of species in this group has proceeded in a different fashion than in the case of einkorns, emmers, and soft wheats. Here divergence has not expressed itself in the degree of chromosome conjugation, but has resulted in the inviability of the hybrid gametes.

In the mountainous regions of Armenia there will no doubt yet be made not a few discoveries which will extend our knowledge of the phylogeny of wheats. Even now we have ample grounds for affirming that precisely here is the center of form-genesis of this fourth group of wheats.

Laboratory of Genetics.
Institute of Plant Industry.
Pushkin.

Received
23. III. 1939.

REFERENCES

[1] М. М. Якубцинер, Соц, раст., 11 (1934). [2] П. М. Жуковский, Зап. Научн.-прикл. отд. Тиф. бот. сада, III (1923). [3] Е. Столетова, Тр. по пр. бот., ген. и сел., 14, I (1934). [4] К. А. Фляксбергер, ibid., XVI, 3 (1926). [5] П. М. Жуковский, ibid., XIX, 2 (1928). [6] А. Л. Декапрелевич и В. Л. Менабде, Научн.-прикл. отд. Тиф. бот. сада, VI (1929). [7] А. Г. Хинчук, Тр. по прикл. бот., ген. и сел., XX (1929). [8] H. Kihara, Proc. of the 5th Pacific Sci. Cong., 4 (1933). [9] F. Lilienfeld u. H. Kihara, Cytologia, 6, 1 (1934). [10] Д. Костов, Тр. Ин-та ген., 11(1937). [11] H. Kihara, Mem. Coll. Agr. Kyoto Imp. Univ., 41 (1937). [12] E. Makushina, C. R. Acad. Sci. URSS, XXI, No 7 (1938). [13] Г. Д. Карпеченко, Теорет. основы селекции, 1 (1935). [14] М. И. Хаджинов, Тр. по прик. бот. ген. и сел., сер. II, 7(1937). [15] J. Yamasaki, Jap. Journ. Bot., VIII, 2 (1937). [16] H. Kihara a. Nishiyama, Lab. Genetics Biolog. in Kyoto Imp. Univ., 2 (1928). [17] В. Н. Лебедев, Явления аутосиндеза в межвидовых гибридах мягкой пшеницы и их значение для проблемы филогенеза пшеницы (1933). [18] М. М. Якубцинер, Тр. по прикл. бот., ген. и сел., сер. V, 1 (1932).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

GENETICS

INHERITANCE OF SEX IN *MERCURIALIS ANNUA* IN RELATION TO CYTOPLASMATIC THEORY OF SEX-INHERITANCE

By D. R. GABE

(Communicated by N. I. Vavilov, Member of the Academy, 23.III.1939)

Mercurialis annua belongs to semi-dioecious plants, i. e. is characterized by incomplete separation of sexes. Individuals of the one sex bear more or less numerous flowers of the opposite sex.

According to Correns (1) and a number of other writers, sex is inherited in semi-dioecious plants in the same manner as in purely dioecious ones: one of the sexes should be homogametic, the other heterogametic. According to the results obtained by Jampolski (9–10), the two sexes, male and female, proved to be homogametic in *M. annua*, i. e. both male and female plants upon self-pollination produced progeny which was identical to them.

Homogamety of the female sex in *M. annua* was noted before Jampolsky by a number of writers (2, 3, 6, 7, 8). As to homogamety of the male sex, data are contained only in the works of Jampolsky who obtained from 283 seeds collected from several tens of self-pollinated male plants 75 individuals, all of which proved to be of male sex. On the basis of these data as well as on variability of sex, Jampolsky advanced a theory, according to which sex in *M. annua* is determined exclusively by the cytoplasm, chromosomal mechanism not being concerned in sex determination.

Inasmuch as data published by Jampolsky are difficult to explain from the viewpoint of theory of sex determination generally adopted in genetics, and the peculiar «no-chromosome» theory of sex determination seems to be insufficiently founded when genetically considered, it appeared to us of importance to study once more in more detail sex-inheritance in *M. annua*, in order to find out whether this plant actually is exceptional with regard to the manner of sex-inheritance.

The work was conducted by us during three seasons (1936, 1937, 1938). In morphology, development and variability of sex our material was similar to that of Jampolsky. The following three combinations were analyzed in the study of inheritance of sex in *M. annua*:

1) ♀ × ♂ (open pollination and artificial pollination);

2) ♀ × ♀ (self-pollination and application of pollen from other female plants);

3) ♂ × ♂ (open pollination, seeds being collected from non-isolated male plants*).

* Jampolsky has obtained seeds from self-pollination of ♂ plants.

We have obtained the following results:

1) ♀×♂; over 20 different families were analyzed. The proportion of male plants varied from 37.0 to 61.5%, the average being equal to 46.7% (from the total of 2831 plants 1507 were ♀ and 1324 ♂). There was thus a 1 : 1 segregation, typical of genotypic sex determination.

2) ♀×♀; 15 families were available from both self-pollination of ♀ and from crossing ♀×♀. The progenies were of female sex in all cases. Male plants constituted a negligible proportion amounting to 0.6%. Out of the total of 505 plants 502 were ♀ and only 3 ♂. The appearance of the latter could be explained by insufficient isolation of the female plants. These data demonstrate the homogametic nature of the female sex in *M. annua*.

3) ♂×♂; progenies of about 30 different male plants were analyzed. In all the families segregation for male and female individuals was observed. Altogether 615 plants were obtained; 179 ♀ and 436 ♂, i. e. the male plants constituted 70.9%.

When the male plants reached maturity and full blooming began, it appeared that two types could be rather easily distinguished among the male plants of this combination. Male plants of the one type which could be called the normal one, possessed anthers of bright yellow colour, dehiscible; good pollen, the proportion of bad pollen grains being usually negligible. Male plants of the other type which should be called anomalous, possessed greenish non-dehiscent or incompletely dehiscent anthers. Pollen was sticky and a considerable fraction of it (40—80%) consisted of anomalous pollen grains. Anthers were not infrequently underdeveloped. In all the remaining characters the two types of male plants were indistinguishable from each other.

Segregation for the two sexual types in the combination ♂×♂ is presented in Table 1.

Table 1

	Number of plants	Numbers			Per cent		
		♀	♂ n.	♂ an.	♀	♂ n.	♂ an.
108_{11} ♂	49	13	25	11	26.5	51.0	22.5
38_{11} ♂, 99_{11} ♂ and oth.	26	7	13	6	26.9	50.0	23.1
728_3 ♂	208	61	104	43	29.3	50.0	20.7
728_8 ♂ self-pollination.	11	2	5	4	18.6	45.4	36.0
76_{21} ♂	44	13	24	7	29.5	54.6	15.9
76_1 ♂	43	21	18	4	48.9	41.8	9.3
The remaining 20 families . .	221	62	107	52	28.1	48.4	23.5
	602	179	296	127	29.7	49.1	21.2

As shown by Table 1 the numerical relation of the three sexual types approaches 1 : 2 : 1, although one should note a clear excess of female plants and shortage in anomalous male ones. The 1 : 2 : 1 segregation could result only from heterozygoty of the male sex, i. e. if it had a *XY* nature causing after self-pollination a segregation 1 *XX* : 2 *XY* : 1 *YY*.

The numerical relations of the three sexual types obtained (♀, ♂ n. and ♂ an.) suggest that normal male plants possess a *XY* genotype and the anomalous ones—*YY*. To prove this, genetical analysis of the two types was conducted following the schemes: ♀×♂ n., ♀×♂ an.

If our hypothesis were correct, the following segregation should be expected

1) ♀ $XX \times$ ♂ $XY \longrightarrow 50\%$ ♀ XX: 50% ♂ XY,
2) ♀ $XX \times$ ♂ $YY \longrightarrow 100\%$ ♂ XY.

Twelve normal and 14 anomalous male plants were analyzed. In Tables 2 and 3 are presented the results according to individual families.

Table 2

♀×♂ n. (1672 seeds planted, 1097 seeds germinated — 65.7%)

	Total	♀	♂	% ♂
♀×108_{10} ♂	13	10	3	23.0
♀×108_{1} ♂	31	12	19	61.3
♀×108_{2} ♂	42	21	21	50.0
♀×730_{1} ♂	90	47	43	47.8
♀×730_{18} ♂	102	102	0	0.0
♀×731_{3} ♂	56	31	25	44.8
♀×731_{10} ♂	82	48	34	42.2
♀×738_{2} ♂	214	118	96	44.8
♀×738_{20} ♂	69	42	27	38.1
♀×739_{4} ♂	57	30	27	47.2
♀×739_{7} ♂	204	93	111	54.4
♀×730_{16} ♂	114	54	60	52.6
	1074	608	466	43.4

Table 3

♀×♂ an. (1211 seeds planted, 683 seeds germinated — 56.4%)

	Total	♀	♂	% ♂
♀×108_{5} ♂	15	0	15	100
♀×108_{8} ♂	21	0	21	100
♀×730_{11} ♂	49	1	48	98
♀×730_{12} ♂	59	4	55	93.4
♀×731_{6} ♂	11	0	11	100
♀×734_{1} ♂	35	2	33	94.7
♀×738_{3} ♂	97	0	97	100
♀×738_{5} ♂	22	0	22	100
♀×738_{7} ♂	164	0	164	100
♀×738_{11} ♂	19	0	19	100
♀×738_{15} ♂	32	0	32	100
♀×738_{17} ♂	57	0	57	100
♀×739_{3} ♂	50	1	49	98
♀×739_{9} ♂	41	5	36	87.8
	672	13	659	98.0

The comparison of Tables 2 and 3 corroborates the hypothesis that normal male plants have the XY formula and the anomalous ones are YY. As a

matter of fact, the progeny of each normal male plant (with the only exception of the plant 730_{18} ♂) consists of female and male individuals, the latter constituting altogether 43.4%. On the other hand, the progeny of the anomalous male plants appear to be purely or almost purely male. Single female plants most probably appeared as a result of insufficient isolation of the female plants during crossing.

All the above data fully prove heterogamety of the male sex in *M. annua*.

The inheritance of sex in *M. annua* thus follows the usual scheme, according to which one of the sexes should be homogametic and the other—heterogametic. In this case the female sex is homogametic *(XX)* and the male sex is heterogametic *(XY)*. The cytoplasmatic theory of sex determination advanced by Jampolsky as an alternative of the chromosome theory thus has no ground.

We failed to detect, it is true, either in somatic plates, or in meiosis among the 16 chromosomes ($2n$) the pair of heterochromosomes typical of the dioecious organisms. This may be explained by the assumption that the *Y*-chromosome has not suffered in *M. annua* any noticeable degradation peculiar to many heterochromosomes. Absence of cytologically discernible degradation of the *Y*-chromosome is wholly in agreement with the fact that the *YY* plants differ but slightly from the normal male plants. Dioeciousness without any more or less pronounced differentiation of the sex chromosomes is quite possible and represents a necessary stage of evolution of *Anthophyta* from hermaphroditism to dioeciousness.

Laboratory of Experimental Systematics.
Peterhoff Biological Institute.

Received
27. III. 1939.

REFERENCES

[1] C. Correns, Bestimmung, Vererbung und Verteilung des Geschlechts bei den höheren Pflanzen (1928). [2] Bitter, Ber. d. deutsch. bot. Gesellsch., XXVII (1909). [3] Krüger, ibid., XXVI (1908). [4] Розанова, Теория основы селекции, I (1935). [5] Рыжков, Генетика пола (1936). [6] Strasburger, ZS. Bot., 1 (1909). [7] Strasburger, Histolog. Beiträge, 7 (1909). [8] Strasburger, Jahrb. f. wissensch. Bot., XLIII (1910). [9] Jampolsky, Am. Journ. of Bot., 6 (1919). [10] Jampolsky, ZS. f. ind. Abst. u. Vererb., LV (1930).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

GENETICS

ON THE METHODS OF ARTIFICIALLY INDUCING POLYPLOIDS BY TREATMENT WITH COLCHICINE

By N. K. KOLTZOFF, Corresponding Member of the Academy

After publication, at the end of 1937, of articles by the American geneticists A. Blakeslee and B. Nebel, intensive research on artificial induction of polyploidy in plants by means of colchicine treatment was carried on in all countries. Colchicine is an expensive and rare preparation; it cannot be obtained everywhere, therefore this method was used much less in the Soviet Union in 1938 than its interest would have justified; the method suggested by Prof. Shmuck to replace the use of colchicine by treatment with acenaphthene fails to give satisfactory results in some species of plants.

Colchicine is a very potent cellular poison. According to Ludford, a colchicine concentration of as little as 0.0001% will kill vertebrate tissue cultures. Blakeslee suggests the use of a 0.2—0.4% solution to induce polyploidy in plants. Is it true that there is such a great difference between vertebrate tissues and flowering plant roots in their reaction to this poison? I decided to investigate this point. I chose for my experiments germinating vetch seeds because their chromosome complex was thoroughly explored by Svieshnikova: their haploid chromosome number is six, and most of them show distinct individuality in metaphase.

By treating vetch seeds with colchicine solutions, it is not difficult to induce polyploidy in the roots. In most cases polyploidy arises not in a few separate cells or sections, but in all or in almost all cells of the germinating root. Probably the action of colchicine is physico-chemical: by penetrating the cell, it changes the viscosity of the protoplasm and hinders the arising of the spindle. Inside the nuclear membrane the chromosomes split periodically during the stage of late prophase; then the nucleus rests. In the case of preparations fixed without washing out colchicine, metaphase and anaphase free from nucleus membrane have a pycnotic appearance similar to that observed by many authors in animal tissues; after washing out the colchicine before fixing, it is possible to find normal metaphase and anaphase. Depending on the length of the action of the poison solution, a very high degree of polyploidy can be obtained; in enormous cells with huge nuclei we can often observe not only $8n$ or $16n$ chromosomes, but also $32n$ and even $64n$. On my slides there are cells, in which the number of chromosomes is much greater than 1000 (probably $256n$). But even in this case such a multipolyploid cell is not the only one on my slide, but is surrounded by gigantic cells with resting nuclei lacking distinct chromosomes. If there are chiefly tetraploid cells in the root, it is in general possible to find di-

ploids and a few octoploids; if the majority of cells are 16-ploids, there are also 8-ploids and 32-ploids. Sections of all the roots which were even a short time in more or less strong solutions of colchicine show obvious signs of poisoning. The cap of dead cells on the surface end of the root and the swelling of the root which was induced by treatment with colchicine are sometimes very thick, up to 10 and more cell layers, which occasionally fall off without disintegrating into separate cells. But under the cap several layers of meristem surface cells also show signs of dying off. Most of them have a disintegrating nucleus; the cell walls are usually abnormally thick. The meristem cells usually show disintegrating and pycnotic nuclei.

I have set myself the task to determine that threshold concentration, at which the traces of colchicine poisoning would be a minimum without eliminating the capacity to induce polyploidy. In May 1938 three series of experiments (*A*, *B*, *C*) were undertaken. For *A* and *B* the following colchicine solutions were used: 0.2—0.1—0.05—0.025—0.01 and 0.005%. When the most dilute of these solutions caused roots with typical polyploid swellings to appear, a third series (*C*) was undertaken, for which 0.005—0.0025—0.0012 and 0.0006% solutions were used. In *A* and *C* seeds previously germinated in water were put into the colchicine solution; in *B* dry seeds were put directly in the colchicine solution. The seeds remained in the solutions *A* and *B* from 1 to 12 days and in the *C* solution from 3 to 9 days. In all there were nearly 100 experiments involving different methods of colchicine application. 15 seeds were used in each of these experiments. At the end of each experiment the germinated seeds were washed with water for several hours to remove the colchicine; then some of them were planted in pots; 2—4 seeds were fixed with Navashin fluid and stained with haematoxylin Heidenhain or Feulgen to prepare a series of slides. In all, more than 200 roots were thus treated and examined.

The slides disclosed a definite difference between the parallel series *A* and *B*. The polyploids among seeds previously germinated in water were more frequent than among dry seeds, both being placed in colchicine solutions of the same strength and for the same time. Among 83 roots examined in *A*, 66 polyploids (79%) were found, and among 92 seeds in *B*—42 polyploids (45.6%). Treatment of previously germinated seeds yields a maximum of polyploids in 24 hours; seeds which have not been previously germinated yield a maximum in three days. Only earlier phases of polyploidy ($\times 4$ and $\times 8$) occur during the first two days; multipolyploids ($\times 16$ and higher) appear after three days; subsequent treatment gradually raises the percentage of the latter, which sometimes reaches 100%.

The influence of the concentration of colchicine is shown in Table 3 which summarizes the results of all the experiments for various periods with previously germinated seeds. This shows that the effectiveness of colchicine treatment varies but little within concentration limits from 0.2 to 0.0012% and even to 0.0006%. In dilute solutions the number of roots with a normal chromosome complex even goes down, sometimes dropping to 0; on the contrary, the percentage of multipolyploids increases. But I do not attach great importance to the absolute number of roots changed in different experiments, partly because of the small number of experiments, and partly because roots for fixation were deliberately selected and not taken at random. In many cases macroscopic changes determined my selection of the roots for fixation; thus, I had unchanged seeds left for planting; this refers especially to the *C* experiments which were to show that polyploids may arise in the most diluted colchicine solutions.

The experiments described by me show that to induce polyploidy in vetch it is not necessary to use strong solutions, 0.2 or even 0.4% as the

Table 1

	Diploids	Tetrapl.	Octopl.	Multipl.	Total	Polypl. (in %)	Multipl. (in %)
Series *A*	17	38	8	20	83	79.0	24.1
Series *B*	50	21	9	12	92	45.6	13.0
Series *C*	1	2	5	18	26	96.2	69.2

Table 2

Polyploidy as a Function of the Time of Treatment According to Series *A* and *C*

Days	Diploids	Tetrapl.	Octopl.	Multipl.	Total	Polypl. (in %)	Multipl. (in %)
1	2	10	1	0	13	84.6	0
2	4	6	0	0	10	60.0	0
3	2	11	5	8	26	92.3	30.8
4	0	5	5	9	19	100	47.4
6	1	4	0	5	10	90	50.0
9	5	2	2	10	19	73.7	52.6
12	4	2	0	6	12	66.6	50

Table 3

Polyploidy as a Function of the Concentration of the Colchicine Solutions for All Treatment Periods (the last 4 rows of figures refer to *C*, in which the roots were first fixed after three days of colchicine treatment)

% colchic.	Diploids	Tetrapl.	Octopl.	Multipl.	Total	Polypl. (in %)	Multipl. (in %)
0.2	5	5	1	0	11	54.6	0
0.1	2	7	1	4	14	88.7	28.5
0.05	6	5	0	2	13	53.8	15.4
0.025	0	11	1	8	20	100	40.0
0.01	4	4	2	0	10	60.0	0
0.005	0	6	3	6	15	100	40.0
0.005	0	0	1	6	7	100	85.7
0.0025	0	0	3	4	7	100	57.1
0.0012	0	0	0	6	6	100	100
0.0006	1	2	1	2	6	83.3	33.3

Americans recommend for datura and for other plants, but it is possible to use solutions 100 or even 200 and 400 times weaker. The weaker the solution of colchicine, the less is its poisonous effect on the general state of the germinated seeds. This is evident not only from the microscopic slides of roots already mentioned, but also from planted seeds germinated in different solutions of colchicine.

Planting conditions were equally unfavourable for all my series; but none of the seeds which had been in 0.2 or 0.1% solutions of colchicine for 24 hours or longer developed into a viable plant. Only a few of the seeds treated with a 0.05% solution germinated, and these chiefly with diploid roots. Beginning with a 0.25% solution of colchicine, the germination of planted seeds approaches the normal, especially in the case of short treatment by colchicine. The lateral roots of plants which rooted well have often only

tetraploid cells after treatment by 0.025—0.0006% colchicine solutions; but roots with normal diploid cells may be found among them; cells of multipolyploid type did not occur. The leaves of growing plants usually had stigmata of normal size, but there were also leaves with very large stigmata, obviously with tetraploid and octoploid cells. A detailed analysis of plants and seeds which have been obtained is not within the aim of the present paper.

Thus, the germinated vetch seeds prove to have nearly the same sensitivity to the poisonous action of high colchicine concentrations as have the tissues of mammalia; but the dilute solutions of colchicine (0.001% and less) induce polyploidy in the vetch roots as definitely as stronger solutions: Immersing young vetch branches in 0.001% of colchicine resulted in the finding of many polyploid mytoses in the buds.

These conclusions can obviously not be extended to other plants; the minimum concentrations of colchicine producing a minimum toxic effect and inducing a sufficiently large percentage of tetraploidy must be determined for each plant.

Institute of Experimental Biology.
Academy of Sciences of the USSR.
Moscow.

Received
31. III. 1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

PLANT PHYSIOLOGY

A CONTRIBUTION TO THE PROBLEM OF PARTHENOGENESIS IN *HUMULUS LUPULUS* L.

By I. N. HOLUBINSKY and M. S. SCHLOSS

(Communicated by N. I. Vavilov, Member of the Academy, 23.III.1939)

The opinion that *Humulus Lupulus* L. possesses a strong tendency to parthenogenetic seed production prevails in literature. An exact answer to the question as to whether parthenogenesis is inherent to *Humulus Lupulus* is of great practical importance. Seed production in hop is very undesirable for beer brewing, since it impares significantly the quality of beer. Many local workers became thus inclined to explain increased seed production in hop by parthenogenetic seed production, following literary data. This discouraged collective farmers in their efforts to destroy male plants around plantations which lead to further increase in seed production.

Experiments on possibility of parthenogenetic seed production in hop have been conducted during the last years at the Ukrainian research station of hop-growing. The first experiments (1929—1934), however, were of a rather accidental character; they were conducted without application of ony definite technique and at a small scale; owing to this evidence produced was conflicting.

Beginning by 1935 we have started cytological investigation on parthenogenesis, and in 1937 arranged rather broad field experiments; as many as possible inflorescences were isolated. In order to ensure best conditions for eventual parthenogenetic development different isolating devices were applied: ordinary parchment bags, glass-cylinders, dense baize and double cheese-cloth. The latter, as shown by experiment, do not fully prevent penetration of pollen, so that they had to be discarded. After the dying off of the stigmata, the isolators were replaced by cheese-cloth for better circulation of air. Examination for seed production was made at the time of technical ripeness. At the same time first maturing hops from early maturing varieties were analyzed; these female plants bloomed earlier than the male ones, so that pollination was entirely impossible even without isolation. In order to find out whether there were any varietal differences in relation to parthenogenesis, experiments were made with six varieties (Serebrjanka, Rogatinsky, Golding, Haag, Clone No. 9 and Clone No. 18).

The results of our experiments are diametrically opposed to those of Kerner (1) and Wettstein (3). Neither in 1937 nor 1938 parthenogenetic seeds were observed. In 869 isolated hops (during two years) only three fruits were found; the appearance of these fruits, however, should be explained by a certain delay of isolation of the one of varieties (Clone No. 9), so that pollen could have reached stigmata prior to isolation. Each hop contains

not less than 20 flowers, so that a very great number of experimental (isolated) ovules gave no parthenogenetic seeds.

Examination of 250 hops of the three earliest varieties (Chuvashsky, Skorospelka and Zeleny Krugly), conducted in order to check the possibility of parthenogenetic seed production, demonstrated complete absence of seeds. This fact, it seems to us, is especially convincing, since conditions were especially favourable to demonstration of parthenogenesis, inasmuch as no seeds were observed in hops formed by these varieties after male plants have begun blooming.

Medvedeva ([2]) arrived at an analogous conclusion from isolation experiments on female plants of hemp. Cytoembryological study gave no positive results during two years. In spite of a significant number of ovules studied (about 600), not a single case could be observed when there was anything reminding parthenogenetic or apogamous development.

The results obtained by us thus disprove the statement made by Kerner and Wettstein on parthenogenesis in hop as being a phenomenon wholly normal for this plant. In six varieties of hop parthenogenetic seeds were observed neither under isolators, nor even after open pollination.

It may be concluded, then, that parthenogenetic seed formation in hop takes place to a very insignificant extent if at all, and does not influence the proportion of seed setting in plantations. Seed formation should be explained by the pollination of the male plants. The more untenable is Wettstein's assumption that just parthenogenetic seed formation causes unequality of sexes when hop is propagated by seeds (excess in female plants). One can hardly agree either with those, who postulate that this phenomenon is of facultative nature or that there are special races inclined to parthenogenesis. The mere fact that all the 40 plants grown by Wettstein from such «parthenogenetic» seeds proved to be of female sex, cannot be considered as proving the correctness of his assumption, since the proportion of male plants is always low in hop, even when artificially pollinated, so that among few plants (40 individuals in Wettstein's case) male individuals could easily be missing.

Research Station of Hop Growing.
Zhitomir.

Received
27. III. 1939.

REFERENCES

[1] Кернер фон-Марилаун, Жизнь растений, 2, История растений (1902). [2] Г. Б. Медведева, Сб. Генетика и селекция конопли, Тр. ин-та конопли, вып. 5 (1937). [3] Wettstein, Flora, 118—119 (1925).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

PLANT ANATOMY

HISTOLOGICAL INVESTIGATION OF LEAF OF *PRIMULA NIVALIS* PALL. AND *CERASTIUM OVATUM* BOISS.

By I. I. CHKHUBIANISHVILI

(Communicated by A. A. Richter, Member of the Academy, 28.III.1939)

The investigation of leaf structure in alpine plants by different workers has led to controversial statements ([1—9] and others). This is possibly because the various species of the alpine flora growing in different stations have been studied in insufficient numbers and no uniform method guided the work.

The principle subject of controversy has been the structure of the mesophyll. The opinion advanced by Bonnier, Wagner and Baranov, for instance, is that the oreophytes possess a strongly developed palisade parenchyma, while Leist, McDougall and others have come to a diametrically opposite view. This is also the case with the questions as to how the cells of the palisade parenchyma

Fig. 1. *Primula nivalis* Pall.

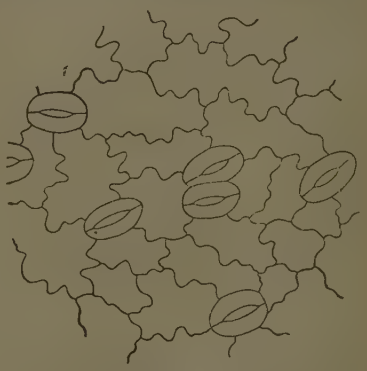


Fig. 2. *Primula nivalis* Pall.— lower epidermis.

are directed to the epidermic surface, how thick is the leaf, how are the stomata distributed in it and so on.

According to Baranov, the isolateral-palisade type of leaf structure predominates in Chimhan (Western Tian-Shan), whereas Lohr has found the dorsiventral type to predominate in the Alps. But these authors have worked at definite heights above sea level, and we have no reason to extend their data over alpine plants growing in other belts.

The subject of the histological study reported in the present communication is the leaf of *Primula nivalis* and *Cerastium ovatum* Boiss., growing

in the snow-bound belt of the Mount Kazbek on the naked rocks of the southern slope, at a height of 4200 m above sea level. Rocky locality, climate of a glacial region.

Primula nivalis Pall. Leaf blade isolateral, spongy of a compact type; thickness up to 102.5 μ (Fig. 1).

Epiderm: 46.3% of the total leaf thickness; cells large, elongated, among them shorter cells occur occasionally, radial wall, strongly sinuous. The upper epiderm thicker than the lower one by nearly 7.5 μ, the thickening of its external wall is a little greater, compared to that in the lower epiderm. Cuticule weakly developed on both sides of the leaf. On their surface the cell walls are characterized by sinuous outlines, the sinuosity being more pronounced in the lower epiderm than in the upper. The number of cells in the upper epiderm is 342 per 1 mm² as against 532 in the lower epiderm.

Stomata: occur on both sides of the leaf (amphistomous leaf), predominate in the lower epiderm, where their number is 133 per 1 mm² as against 36 in the upper. The length of the upper closing cells attains 50 μ as against 45 μ in the lower. They are level with the epiderm, but for their being somewhat lifted up. Typical stomata twins are frequently encountered in the lower epiderm, when two stomata are drawn together so that their coats come in contact. The stomata are in contact both at the lateral and foreside (Fig. 2).

Mesophyll: not differentiated, consists of fully homogeneous parenchymous cells, not very large in size. Cells, elongated in the direction of epiderm, occasionally roundish. Tissue compact; intercellular spaces weakly developed, their total volume does not exceed that of the cells.

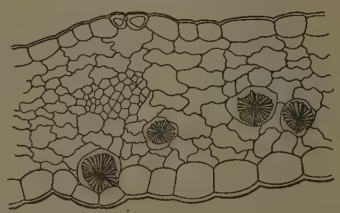


Fig. 3. *Cerastium ovatum* Boiss.

Cerastium ovatum Boiss. Leaf blade isolated, spongy of a compact type; thickness up to 132.5 μ (Fig. 3).

Epiderm: 33.9% of the total leaf thickness. Cross-sections show elongated cells, among which roundish ones are of no seldom occurrence. The upper epiderm by 10 μ thicker than the lower; the thickening of external wall in the former by 1.3 μ greater than in the latter. Cuticule weakly developed. The outward appearance of the cell walls presents strongly sinuous outlines; the sinuosity is equally expressed on both leaf sides. The number of cells is 209 and 304 per 1 mm² in the upper and lower epiderm respectively.

Stomata: occur on both leaf sides, but their number is greater on the lower one: 95 as against 76 per 1 mm². Length of closing cells in the lower and upper epiderm is equal and attains 42.5 μ. They are level with the epiderm, though markedly lifted up on the under side, compared to the neighbouring cells.

Mesophyll: not differentiated; consists of fully homogeneous large parenchymous cells. The latter are elongated parallel to the epiderm; cell walls thin, sinuous and the cells, therefore, are of no definite shape. Intercellular spaces developed, but not to such an extent as to exceed the cells in the total volume. In mesophyll the cells with large druses are of frequent occurrence, they are often met with in the upper epiderm.

The following may be pointed out in conclusion. Bonnier has suggested that the development of palisade tissue in plants attains its maximum at a certain altitude above sea level and then goes down as we rise higher. His thought may be developed farther by an assumption that the decrease in

the development of palisade tissue finishes with complete disappearance of the latter from the leaves of the plants growing on very great mountain heights. This assumption is substantiated by the investigation of McDougall and the histological description here referred to.

Tbilissi Chair of Vegetable Physiology and Anatomy.
Georgian State Agricultural Institute L. P. Beria.

Received
25. III. 1939.

REFERENCES

[1] П. А. Баранов, БСАГУ, вып. 8 (1925). [2] Karelshchikov, Bull. Soc. Nat. (1866). [3] Э. Струмнєр, ГПОЕ, XXIX, вып. 3 (1899). [4] G. Bonnier, Rev. gén. de botan., **12** (1890). [5] G. Bonnier, Ann. d. Sci. Nat. botan., Sér. 7, XX (1895). [6] K. Leist, Mitteil. d. Naturw. Ges., Bern (1889). [7] P. L. Lohr, Rec. d. Trav. botan. Néerlandais, XVI, livr. 1 (1919). [8] Mc Dougall, Plant Ecology (1927). [9] A. Wagner, Sitzungsber. d. K. Akad. d. Wissensch. Wien, Math. Naturw. Klasse, **101**, Abt. 1 (1892).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

PLANT ANATOMY

CONDITION OF STARCH IN POTATO AT DIFFERENT STAGES OF TUBER RIPENING

By F. KH. JAGER and M. S. JAKOVLEV

(Communicated by A. A. Richter, Member of the Academy, 28.III.1939)

The starch of potato is one of those most important products which find extensive and manifold application in the economy of this country. But in spite of its great importance, there are many questions about it which have not been satisfactorily elucidated up to the present. Such appear to be the formation and growth of starch grains in different potato varieties, the changes taking place on the storage of tubers and the nature of the dormancy period.

While searching for early potato varieties resistant to cancer we made experiments, whose object was to study the yield of certain varieties with reference to the time of harvest(²). In this connexion we had to trace out the process of starch accumulation in the tubers and to establish the relation between the yield and the condition of starch grains in different varieties of potato.

The starch was studied in concentrated solution of sucrose without staining, whereby iodine as potassium iodide was made use of. When I + IK is applied according to usual method, iodine will not be held for a long time in the reaction with starch and is soon volatilized. Quite different picture is observed, when a concentrated solution of sucrose is used. Here the colour of iodine is retained for months and even for years. While examining the preparations made eighteen months ago, we could state that most of them have well preserved their colour. However, the varieties tested showed varying degrees of ability to retain the colour of iodine in the concentrated solution of sucrose. The early varieties retained colour somewhat better than the late ones. Yet, these differences were observed only at early stages—in the tubers harvested July 7 and, partly, July 17. When harvesting was done at a later time, no such differences could be detected. This should be taken to mean that the chemical properties of the starch are changing during the growth and formation of its grains.

In their formation the starch grains as a product of vegetable cell's life activity are in the first instance bound up with plastids. No deposition of starch is known outside of the activity of the latter. The history of appearance

and formation of starch granules has been traced out by a number of workers.

The reserve starch in potato tubers is far from being homogeneous morphologically. The greatest variety of starch granules with regard to size is encountered in selection strains as compared, for instance, to semi-cultivated or south-american forms of potato.

Growth and formation of starch grains proceed also differently in different varieties. A very quick growth is observed in early varieties as Di-Vernon, Early Rose and others. Their tubers exhibit great amounts of large roundish starch grains, as early as in the first half of July. Toward the end of this month they may be considered as having completed the growth and formation of starch grains, since if harvested later they do not show any appreciable changes in the tubers.

Similar picture in the accumulation of starch is displayed by the varieties which are intermediate with regard to the time of ripening (Witch Hill, Paul Wagner, Great Scot and others). Toward the end of July considerable amounts of coarse-grained starch may be observed in them. But it is only in the middle of August that the grains attain their optimal size. An especially quick growth of starch grains it observed in the variety Witch Hill which has this process completed at the beginning of August (Fig. 1). Variety Great Scot, though it belongs to intermediate ones, differs nevertheless from the others in its having smaller cells and not very coarse starch (Fig. 1). Coarseness of starch is conspicuous in variety Paul Wagner. The latter toward the middle of August has a great many large grains of roundish or angular shape.

Fig. 1.

In later varieties (Jubel, Rosafolia, Isolde, hybrids Di-Vernon × Centifolia and Fürstenkrone × *S. f. tocanum*) the accumulation of plastic substances and among them of starch grains proceeds more slowly. If harvested in July, the tubers possess great numbers of small and medium-sized grains. In the middle of August the latter predominate and there occur single large grains along with reasonable amounts of fine grained starch. Hence, the formation and growth of starch are not yet completed toward that time. Late varieties contain, moreover, large amounts of starch grains in the cells of their tubers. It is the presence of those starch grains in the cells and the longer vegetation period which probably account for the fact that the late varieties give higher yield than the early ones.

Among hybrids the greatest interest is due to Di-Vernon × Centifolia which as early as toward the middle of August stores up large amounts of coarse-grained starch.

The degree of ripeness of a potato tuber is to a considerable extent dependent upon condition and ripeness of its starch grains. Tubers which have

reached full maturity are characterized by great quantities of coarse-grained starch with pronounced layered structure and presence of grains with angular outlines. At the early stages of development neither the layers nor angular grains can be distinguished. Both these characters become more and more pronounced as the plant increases in age and they appear first in large grains.

Earliness or lateness of potato is correlated with the rate, at which starch is being stored up during the period of vegetation. In early varieties the growth and formation of starch granules proceeds very quickly; in the late ones this process is slowed down considerably and therefore ripeness is achieved later.

Different species and varieties of potato are not alike in their dormant habit. Some of them have the tubers sprouting at once after harvest, while others go through a long period of dormancy (1). The sprouting ability of a tuber is dependent upon a number of factors. Only physiologically ripe tubers can produce sprouts from their eyes, when appropriate conditions are available.

In view of such pronounced differences in the dormant habit one can expect that there will be no equality between the quickly and the slowly sprouting varieties as regards the structure of their tubers and the development of starch granules. Corroborative evidence of this assumption has been furnished by our observation of tubers of different varieties. At two opposite ends of the scale are the quickly sprouting variety Epicur and the slowly sprouting Silesia respectively.

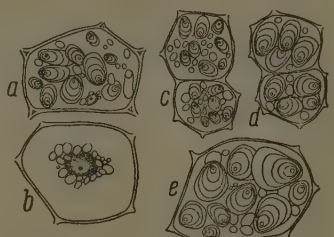


Fig. 2. *a*, *d*—*S. andigenum f. tocanum*; *b*, *c*—*S. ang. f. globosum*; *e*—Epicur.

Most striking differences in the dormant habit have been observed in *S. andigenum*. Among various forms of that species *f. tocanum* has well sprouting tubers, while the tubers of *f. globosum* produce only poorly developed sprouts.

We have noticed in our investigations that the starch grains are larger in the tubers of well sprouting varieties. Accordingly, the variety Epicur has much larger grains compared to variety Silesia. The behaviour of the starch grains reveals especially sharp differences when *f. tocanum* and *f. globosum* are compared. These two forms were investigated in more detail. While comparing the condition of cells in the region of sprouting eyes in tubers of different dormant habit, we observed pronounced differences in the character of formation of starch grains, on the one hand, and in the behaviour of the nuclei, on the other (Fig. 2, *a*, *b*).

The size of the nuclei in the tuber cells is strongly reduced at the base of young sprouts. It would appear that the process of nucleus disintegration is taking place, being attended by an opposite process, the activation of plastid apparatus. The appearance of chondriosomes and afterwards of plastids should be linked up in the first instance with the vegetative life of the nucleus itself. Then more light will be thrown upon those phenomena also which are connected with the conversion of cell contents and release of potato tubers from dormancy.

As regards the structure of young sprouts, the well sprouting and the poorly sprouting tubers differ from each other not only in the structure of their nuclei, but also the character of starch development is somewhat different in these forms. In the tubers of poorly sprouting potatoes the starch grains are smaller (Fig. 2, *c*, *d*, *e*).

It will be obvious that the character of development of starch grains and the conditions of their formation are of great significance for the sprouting of tubers. Thereby the condition of nuclei and the activity of plastids should play an essential rôle in such a complicated process as dormancy.

Laboratory of Plant Anatomy.
All-Union Institute of Plant Industry.
Pushkin.

Received
28. III. 1939.

REFERENCES

[1] Букасов, Теорет. основы селекции растений, 3 (1937). [2] Веселовский, Зап. Ленингр. Плодоовощн. ин-та, 3 (1938).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

ZOOLOGY

ON THE METHODS OF ESTIMATING THE STOCKS OF THE BLACK SEA DOLPHIN (*DOLPHINUS DELPHIS PONTICUS* BARABASCH)

By S. E. KLEINENBERG

(Communicated by I. I. Schmalhausen, Member of the Academy, 29.III.1939)

The fishery for dolphins in the Black Sea is developing very intensively. The catches are increasing from year to year; thus, in our waters the take of 1938 was by 40—50% greater than that of 1937. At the same time we have no data on the stocks of dolphins in the sea; neither do we dispose as yet of an adequate method for their direct quantitative evaluation. Yet a knowledge of the condition of the stock is necessary in order to ascertain whether the fishing exerts a depleting influence on the stock.

One of the methods of obtaining such a knowledge is furnished by the analysis of the age and sex composition of the catches during several years. As the shoals of dolphins are usually of a mixed composition, we can assume that an adequate sample (as shown by a catch) is representative of the age and sex composition of the herd. But even discarding this assumption, the analysis of catches for a series of years can show us: 1) the trend of the age and sex composition of the catch during the year, and 2) the trend of the age and sex composition of the catch for several years. If the former has

Table 1

Year	Months	Number of individual measurements with sex determinations
1933	July, August, September, October	5 815
1934	March, April, May, June, July, August	20 830*
1935	May, June, July, August, September, October	11 628
	Total for three years . . .	38 273

*Moreover, in February 1934 sex determinations were made for 83 animals, without body measurements.

no direct bearing on the condition of the stock, the latter permits to obtain some indices in this respect.

The works by Wheeler (1) and, especially, by Laurie (2) on the ages of the females of blue whales and the effect of whaling upon the stock show that an increase of the number of sexually immature females and a corresponding decrease of sexually mature females in the catches can be taken as a proof of the decline of the stock itself.

We have analyzed the following material obtained during 1933, 1934 (by the expedition of VNIRO)* and 1935 (by the expeditions of ASCHERNIRO)**, month by month.

In determining the age categories, we distinguished only two groups of animals, for males as well as for females—the group of sexually mature and the group of sexually immature animals.

According to the work of Nikolsky and Trüber, we established for these groups the following linear body measurements:

sexually immature males	up to 165 cm
sexually mature males	166 cm and more
sexually immature females	up to 155 cm
sexually mature females	156 cm and more

As this study on the dolphin is the first of its kind and covers only a short period of time, it cannot reveal any great changes in the composition of shoals. Its only object is to develop a serviceable procedure and to obtain comparable material for future investigations.

Simultaneously with our work, an analogous investigation was carried out by Zalkin, but with results totally opposite to ours, since the figures obtained by the author failed to disclose any similarity in the composition of catches for different years. However, Zalkin's methodic is lacking due accuracy and this should probably account for his conclusions which we consider to be erroneous. Let us note the main methodic imperfections of the work.

1. In analyzing the age composition, the author subdivises the sexually immature part of the herd into three year-groups, whereas all the mature animals are united into a single age group.

2. In operating with these age groups, the author often omits to divide them according to sex. Such procedure is obviously incorrect, since females reach sexual maturity earlier than males, so that, consequently, the ratio of sexually mature and sexually immature groups will be different for males and females.

3. Comparing the «yearly» catches, the author does not take into account that the material for different years was not collected at the same periods, although, as we know, the age and sex composition of the catches is not homogeneous during the year.

Now let us pass to the discussion of the results of our work. During the greater part of the year, the sex ratio in the catches is 1 : 1 (II, III, VII, VIII, IX and X months), but in April the relative number of females drops to 32%. In May the ratio increases to 41%, in June to 45%, to reach in the following its former value of about 50%. This phenomenon can, probably, be put down to the departure of females from the fishing grounds in April, May and June. For the same period an increase of the relative number of sexually immature individuals of both sexes is to be noted in the catches. Thus, we see that the age and sex composition is not homogeneous throughout

---

* VNIRO—Institute of Marine Fisheries and Oceanography of the USSR.

** ASCHERNIRO—Asov-Black Sea Institute of Marine Fisheries and Oceanography.

the year and, consequently, when comparing catches for different years, similar calendar periods of time should be used, if the material has been collected only for a part of the year at irregular periods. In our case such periods are: 1933 and 1935—VII, VIII, IX and X; 1934 and 1935—V, VI, VII and VIII.

The figures referring to the sex composition of the catches for the same month of different years are in close agreement and this fact speaks for the homogeneity of the sex composition of the total catches for different years. This is clearly shown in Table 2.

Table 2

Years	Months	Relative number of females, %	Years	Months	Relative number of females, %
1933	VII—X	52	1934	V—VIII	46
1935	VII—X	50	1935	V—VIII	47

Table 3 gives a similar comparison of the age and sex composition.

Table 3

Sex	Age groups	V, VI, VII, VIII		VII, VIII, IX, X	
		1934, %	1935, %	1933, %	1935, %
Males	Sexually immature	29	26	23	25
	» mature	25	27	25	25
Females	» immature	16	18	16	22
	» mature	30	29	36	28

It may be seen that: 1) during 1934 and 1935 the composition of the catches according to age and sex was almost identical; the fluctuations in the numerical strength of individual groups did not surpass 2—3%; 2) sexually immature females were represented in the catches by lesser relative numbers than the corresponding groups of males.

It is obvious that in a population of dolphins this group will also be lesser, inasmuch as females reach maturity earlier than males.

The age composition of males for 1933 and 1935 is practically identical, while the age composition of females is subject to variation. Thus, the relative number of sexually immature females increased in 1935 by 6%, while the number of sexually mature females decreased by 8%.

Table 4 shows the age composition of the catches for each group separately (1934—1935 and 1933 and 1935).

Tables 4 fully confirms the conclusions reached on grounds of Table 3.

Summarizing all these data (Tables 3, 4) we note that the age composition of the males varies but insignificantly in different years, whereas the age composition of the females is subject to much greater variations (Table 4), its changes showing a definite tendency to a decrease of the relative number of sexually mature and an increase of sexually immature females in the catches.

Table 4

Sex	Age groups	V, VI, VII, VIII		VII, VIII, IX, X	
		1934, %	1935, %	1933, %	1935, %
Males	Sexually immature	54	49	49	51
	» mature	46	51	51	49
Females	» immature	35	39	31	44
	» mature	65	61	69	56

However, it would be premature to take this trend for a proof of the depletion of the stocks of dolphin, since our work covers but a short period of time. It is therefore necessary to continue these studies, since the comparison of the results of further investigations with our data will disclose the changes, occurring in the composition of the catches.

In estimating our method we note that: 1) the figures of the age and sex composition of the catches in the different years of the period 1932—1935 agree closely with one another; 2) for the most part of the year, males and females are represented in the catches in identical numbers; 3) sexually immature females are represented in the catches by the least relative numbers.

All the above said leads to the conclusion that the composition of the catches is representative of the age and sex structure of the herd.

Thus, the method used here for the analysis of the catches can be recommended for the control of changes occurring in the composition of the stock. In this way we may obtain an approach to the determination of the condition of the stocks of the Black Sea dolphin.

Institute of Marine Fisheries
and Oceanography.

Received
28.III.1939.

REFERENCES

[1] F. G. Wheeler, Discovery Reports, IX, 351—372 (1934). [2] A. H. Laurie, ibid., XV, 223—284 (1937).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

EXPERIMENTAL MORPHOLOGY

MORPHOLOGICAL RESPONSE OF THE TRANSPLANTED HYPOPHYSIS TO CASTRATION AND THYROIDECTOMY

By K. Z. KAN

(Communicated by I. I. Schmalhausen, Member of the Academy, 29.III.1939)

The studies of Hohlweg and Junkmann (3) and the later study of Martins (6) have shown that typical castrational alterations do not appear in the anterior lobe of the hypophysis transplanted to an animal which is subjected to castration. The structure of the implanted hypophysis remains normal. On the ground of this observation Hohlweg built his hypothesis of the existence of a nervous sexual centre which controls the gonadotropic secretion of the hypophysis depending on the level of sexual hormones. Castrational alterations arise in the hypophysis only if the nervous connexions are intact, when the nervous centre liberated from the depressing influence of the gonads stimulates the hypophysis towards an excessive production of the gonadotropic hormone; the result of this is a hypertrophy of the basophile apparatus of the anterior lobe. On the other hand, Martins (6) thought that the hyperfunctional condition characteristic of the castrate's hypophysis cannot arise under the unfavourable conditions of the transplanted organ. Consequently, according to him, there is no need to assume the existence of a special nervous sexual centre for the explanation of the maintenance of the normal structure by the hypophysis transplanted to a castrated animal.

Of late growing attention is being paid in the world literature to the rôle of the nervous system, particularly of the intermediate brain, in the control of the hypophysary-gonad correlations. Jores (4) summing up the results of the experimental studies of the recent years formulates the following thesis: «The anterior lobe of the hypophysis is the switch shunting nervous impulses into hormonal ones». In regard to the hypophysary-gonad correlation, the researches of Westmann and Jakobson (7) have shown with absolute conclusiveness that the stimulation issuing from the gonads proceeds through the vegetative nervous system to the hypophysis and there is shunted over to the hormonal route. One can expect a similar correlation between the hypophysis and other endocrine glands, i. e. the transmission of the stimulations from these glands through the respective nervous paths to the hypophysis which responds by the secretion of the corresponding gonadotropic hormone.

It is known that the histological structure of the anterior lobe of the hypophysis changes depending on the variations in the functional conditions of the endocrine glands. Collin (1) was the first to show that the nervous connexions play a definite part in these changes. He described a speedily

developing chromophobia in the anterior lobe of the hypophysis of the rabbit after the resection of the ganglion cervicalis superior of the sympathic trunk; this chromophobia, however, disappears after some time. Zeckwer [9] has shown on rats that the disappearance of eosinophile cells after thyroidectomy bears no connexion to the injury of the ganglion cervicalis superior, as this might be assumed on the ground of Collin's work, since after a resection of the ganglion unaccompanied by thyroidectomy the eosinophiles do not disappear. It seemed of interest for the purpose of establishing whether the nervous system plays any part whatever in the realization of these alterations, to study the morphological picture of the hypophysis of the thyroidectomized animals under the conditions of transplantations, i. e. lacking every nervous connexion.

We conducted our researches on male and female rats, in which the histological alterations after thyroidectomy are particularly vivid and consist in the disappearance of eosinophiles and the appearance of the cells characteristic of thyroidectomy [8,5].

Our first series of experiments (19 rats) was a repetition of Hohlweg's and Martins' experiments. We repeated them, because the authors had paid too little attention to the histological structure of the transplant, stating merely that castrational alterations were absent. In our experiments fragments of the anterior lobe of the hypophysis were implanted into the anterior eye chamber of rats, which 7—10 days later were castrated. Grafts were also made from castrates to castrates and to normal rats. The rats of the first group ($N \rightarrow C$) were killed 30—60 days after castration and those of the groups $C \rightarrow C$ and $C \rightarrow N$—14—21 days after the transplantation. The fragments of the hypophyses in these cases were taken from 85-day castrates. In the grafts of hypophyses from normal rats to castrates the transplant differed in nothing from the controls ($N \rightarrow N$, Fig. 1). Excellent vascularization, preservation of the topography of tissue tracts, mitoses, the presence of all the three types of cells, which 1—1.5 months after transplantation show already a tendency towards dedifferentiation, the basophiles disappearing first and the eosinophiles remaining somewhat longer—all these characteristics were common for both the control and the test graft. Castrational cells were not found in the transplants, while the hypophyses of the host showed the typical picture of castration. In the $C \rightarrow N$ and $C \rightarrow C$ experiments we were able to detect, 14 days after transplantation, obvious traces of castrational alterations, which consisted in hyperaemia (even macroscopically the transplant showed a dark-red colour) and in the presence of hypertrophied basophile cells with a clearly visible macula (the site of Golgi's apparatus), though in small numbers (1—3 cells in a section).

21 days after transplantation the traces of castrational alterations were already hardly distinguishable, hyperaemia was absent, isolated large basophile cells, which might be regarded as remnant «castrational cells», stained a pale colour and were hard to distinguish from chromophobe cells. These basophile cells partly suffered pycnosis and degeneration. This series of experiments fully confirmed the results of Hohlweg and Martins that under the conditions of transplantation no castrational alterations arise or are maintained in the hypophysis. According to the latest data of Desclin [2], the number of basophile cells in the transplant is increased already 15 days after castration. Our observations began at least 30 days after castration, but we did not note an increase of the basophiles in any case. Even if Desclin is right, then it must be that they disappear quickly and that these basophiles do not transform into castrational cells.

Our second series of experiments (24 rats) was also divided into three groups: 1) grafts of hypophyses from normal rats (80—100g) to rats of the same weight, which were subjected 5—10 days later to thyroidectomy; these

rats were killed 30—45 days later; 2) grafts of hypophyses from thyroidectomized rats (35—50 days after thyroidectomy) to normal ones; the rats were killed 8, 14 or 21 days after transplantation, and 3) grafts of hypophyses from thyroidectomized rats (35—50 days after thyroidectomy) to thyroidectomized rats (of the

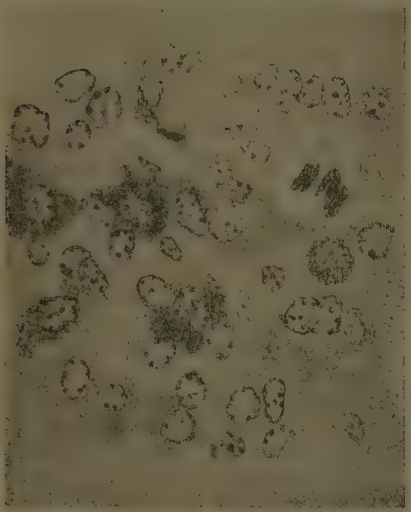


Fig. 1. Transplant of ant. lobe of hyp. $N \rightarrow N$; haem.+eosin; oc. 18x, obj. 40 D, Zeiss (42nd day after transplantation.)

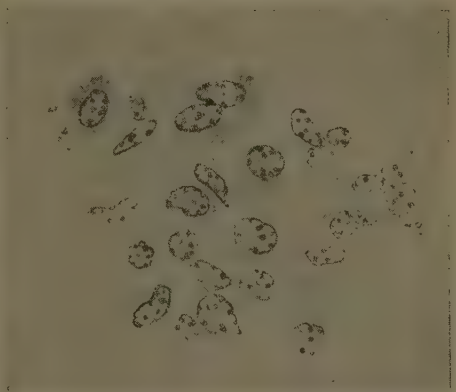


Fig. 2. Transplant of ant. lobe of hyp. $N \rightarrow T$; oc. 18x, obj. 40 D, Zeiss (37th day after transplantation).

same date of thyroidectomy); these rats were likewise killed 8, 14 and 21 days after the graft.

The study of the histological structure of the transplants in the first group ($N \rightarrow T$, Fig. 2) showed that , while the host's hypophysis responds

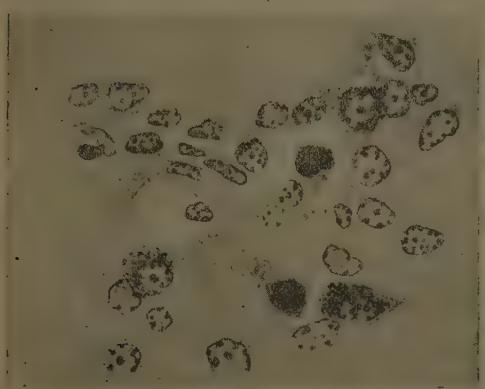


Fig. 3. Transplant of ant. lobe of hyp. $T \rightarrow N$; oc. 18x, obj. 40 D, Zeiss (21st day after transplantation).

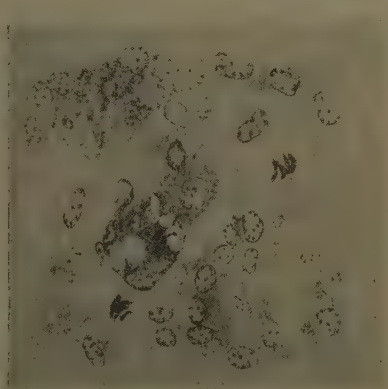


Fig. 4. Transplant of ant. lobe of hyp. $T \rightarrow T$; oc. 18x, obj. 40 D, Zeiss (14th day after transplantation).

to the resection of the thyroid gland by typical alterations, the implant responds in a somewhat peculiar way. We were unable to observe cells of thyroidectomy in any of the cases, however, the eosinophile cells 45 days after

transplantation showed a greater dedifferentiation than that which was visible in the control transplants of the same data of transplantation. The eosinophile cells were very few, they stained quite pale, we succeeded in detecting them only by means of the eosin stain; asocarmine did not stain the plasm of these cells. No complete disappearance of the eosinophiles could be observed in any of the cases.

Of particular interest to us were the two last test groups, since the question naturally arose, whether a restoration of eosinophile cells is possible in a hypophysis deprived of nervous connexions. It was found that in the grafts of fragments of hypophyses from thyroidectomized rats to normal ones ($T \rightarrow N$, Fig. 3) eosinophile cells appeared to the number of 1—2 in a section already by the 8th day after transplantation, while by the end of the 3rd week the preparations showed considerable numbers of eosinophiles. The eosinophile cells were evidently formed from chromophobe cells, since the transplanted fragments of the hypophysis entirely lacked eosinophile cells. In the grafts of hypophyses from thyroidectomized to thyroidectomized rats ($T \rightarrow T$, Fig. 4) eosinophile cells did not reappear, and it was not possible to detect them either on the 8th or on the 20th day after transplantation. If the thyroidectomy of the host proved incomplete, which could be judged on the ground of the presence of eosinophiles in the intact hypophysis and on that of finding fragmental remnants of the thyroid gland at the site of the operation, eosinophile cells appeared in the transplant. As to the cells of thyroidectomy, they were preserved in the transplant at the 8th day after transplantation, at later dates they degenerated irrespective of the condition of the host, i. e. both when it was a normal or a totally or partly thyroidectomized rat. The cells of thyroidectomy degenerated by means of a nuclear pycnosis and of a complete granular disintegration of the plasm. As to hyperaemia which is characteristic of the hypophysis after thyroidectomy, we failed to observe it in any of the transplants.

We see thus that the study of the histological structure of the transplanted hypophysis under the conditions of thyroidectomy of both host and donor shows that the appearing of the cells of thyroidectomy, similar to what we observe in the case of the cells of castration, is definitely bound with the intactness of the nervous connexions of the hypophysis. The differentiation of the eosinophile cells, on the other hand, depends on the presence of the thyroid gland and may arise under the conditions of transplantation only in the case if the thyroid glandular tissue of the host has been preserved.

Received
29. III. 1939.

REFERENCES

[1] R. Collin et L. Hannequin, C. R. S. B., XXI, № 13 (1936); CXX, (1935). [2] L. Desclin, Les hormones sexuelles (1938). [3] W. Hohlweg u. K. Junkmann, Klin. Wochenschr., № 8 (1932). [4] A. Jores, ibid., № 20 (1938). [5] Н. С. Лебедева, Арх. анат., гист. и эмбр., **15**, № 4 (1936). [6] T. Martins, C. R. S. B., 123 (1936). [7] A. Westmann u. D. Jakobsohn, Act. obst. Gyn. Scandin. (1936), quoted after Endokrin., **19**, H. 3/4 (1937). [8] J. Zeckwer, Amer. J. Med. Sci., **190**, 145 (1935). [9] J. Zeckwer, Proc. Soc. exp. Biol. and Med., **35**, № 4 (1937).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

MICROBIOLOGY

METHODICS OF BREATHING DETERMINATIONS IN MICROBES

By L. A. IVANOV, Corresponding Member of the Academy, and E. A. STERN

The determination of breating in microbes demands the use of the most intricate methods involving the pumping of air through a number of containers. According to the scheme proposed by V. L. Omeliansky, it is necessary to have a well working aspiration or water pump and nine containers for each culture. Moreover, in the case of microorganisms, the optimum of the development of which lies above the usual room temperature, it is necessary to have at one's disposal a thermostatic room or a special incubating apparatus, because the volume of the entire mounting renders work with incubators of the usual type impossible. Besides this the work is complicated by the choice of suitable containers for the cultivation of the microbes. Good aeration is necessary for the normal development of the cultures and for the precise determination of the amount of CO_2 liberated by them. To provide sufficient aeration Omeliansky applied in his tests for determining the breathing of Azotobacter special flat containers (height 3 cm).

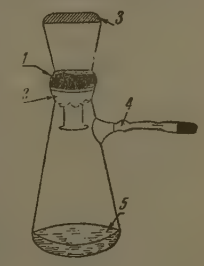


Fig. 1.

In view of these difficulties we worked out a different method for breathing determinations in microbes. This method due to its simplicity is available for every microbiologic laboratory and permits to conduct simultaneous tests with a great number of cultures. It is based on the principle of the binding of CO_2, liberated by microbes, in a closed space by barium hydroxide.

Two conic flat-bottomed flasks are taken: a smaller one for the cultures and a larger one for an air reserve and for the introduction of barium hydroxide. The latter has at its side in the upper part an open offset tube *. Their volume is determined. In our tests with the determination of the Azotobacter's breathing, the volume of one flask was 400 cm^3, that of the other 1100 cm^3. The agar-containing nutrient substrate is poured into the upper (smaller) flask (Fig. 1). After sterilization, when the agar has become solid, the medium is seeded with a suspension of bacteria in water. When the suspension** dries up, the flask is turned upside down and its neck is introduced into the

* Conical flasks, used commonly for filtration under decreased pressure, are suitable for this purpose.

** To quicken the drying up of the suspension, the flask may be placed for 1—2 hours into the incubator.

wider aperture of the lower flask. The place of the joining of the two flasks is wadded with cotton and the border is stopped with plastelin or a special putty, consisting of 2 parts of bee wax and 1 part of swine lard (by weight). This putty does not let the air through and, moreover, allows to separate the two flasks quickly and without difficulty. For the isolation from atmospheric CO_2 a small tube containing sodium lime or a rubber tube with a Bunsen lock*, which has been widely used by Buchner in his well-known researches on fermentation, is attached to the offset. The lock allows the air to come out of the flask in the case of a raised pressure inside it, but does not let the outside air into the flask, if this pressure sinks. The breathing itself in the presence of the absorbing substance (barium hydroxide) is accompanied by a decrease of the volume of air in the flask, due to the consumption of oxygen. On the other hand, the volume of gas is increased when the flasks are placed into the incubator. An approximative computation shows that the volume of air lost in this operation, even if its CO_2 content is raised by breathing to 0.5%, may contain no more than 0.2 — 0.3 mg of CO_2. A direct test with a flask placed into the incubator for 24 hours at 37.5°C and then returned back to room temperature showed a loss of 0.14 mg of CO_2.

The bottom of the lower flask is covered with titrated barium hydroxide water, coloured by phenolphthalein. By the colour of the solution it is possible to judge of the degree to which barium hydroxide is neutralized by the liberated O_2, and this permits to stop the experiment in time, i. e. when there still remains an excess of barium hydroxide.

For our experiments with Azotobacter we take 100 cm^3 of barium hydroxide water containing 7 g of crystallized barium hydroxide per 1 liter of distilled water. The CO_2 liberated by the microbes, being heavier than the air, gathers at the bottom of the lower flask, unites with barium hydroxide and forms a precipitation of barium carbonate. The disposition of the flasks does not permit CO_2 to accumulate at the surface of the culture. After a certain time, e. g. after 3 hours, the flasks are separated. The flask with barium hydroxide is closely stopped by a cork, sealed with paraffin and titrated either immediately or after some time. The flask with the culture of microbes is quickly transferred on another (reserve) flask with fresh barium hydroxide water. Thus, the absorption of carbon dioxide is almost uninterrupted.

Simultaneously with the test flasks similar control ones,without cultures, are set for the purpose of determining the amount of CO_2 contained in the flasks before the test. This amount varied, depending on the CO_2 content in the air of the laboratory, between 0.8 and 1 mg. Barium hydroxide water is usually titrated against hydrochloric or oxalic acid (2.8636 g of pure recrystallized oxalic acid per 1 l of distilled water; 1 cm^3 of this solution of oxalic acid corresponds to 1 mg of carbon dioxide; if a weaker solution is taken it is possible to determine even hundredths of milligrams). The amount of barium hydroxide neutralized by CO_2 is calculated on the ground of the difference between the initial titre and the titre obtained after the test, and after the obtained figure is corrected by subtracting the amount of CO_2 in the control, we obtain the amount of carbon dioxide liberated by the microbes in breathing.

This method provides the possibility to determine the breathing of microbes in a very simple way for any desired interval of time on a large number of cultures.

The results of the tests made according to this method were quite satisfactory. The curve of breathing of the Azotobacter, obtained by the proposed

* The Bunsen lock consists of a black rubber tube, 5 cm long and with 0.5 mm thick walls. One end of this tube is put on the offset tube of the flask, while the other is stopped by a glass rod. In the middle part of the tube along its wall a 1 cm long cut is made with a very sharp knife.

method, does not differ appreciably from the curve of Omeliansky, who used an intricate method involving the pumping through of air.

In spite of the fact that Azotobacter belongs to those microbes which liberate carbon dioxide most intensely being at the same time extremely sensitive to insufficient aeration, the development of the cultures and the formation of pigment by them proceeded in a perfectly normal way. Evidently the aeration proved sufficient, because the liberated CO_2 sank down and was continuously absorbed by the barium hydroxide, and the oxygen content, as a simple calculation shows, could not diminish by more than on third, the latter being possible only in the extreme case, when the flasks are not changed for 3 days and the amount of CO_2 formed in the course of this time reaches 200 mg (=111 cm^3). The formation of such an amount of CO_2 requires the same volume of oxygen (111 cm^3), the most probable respiration coefficient equalizing 1. As 1.5 l of air in the flasks originally contained 315 cm^3 (1500 $cm^3 \times 0.21$) of oxygen, an excess of 204 cm^3 will consequently be left, making up about 14% which is quite sufficient for the breathing of microbes.

Fig. 2.

By the end of the test (24 days after it was started) the microscopic examination of the Azotobacter cultures showed that they remained uncontaminated by foreign microbes, even if the flask's neck was not stopped by a cotton plug. Besides, the test showed also that such a cotton plug does not interfere in the absorption of carbon dioxide by barium hydroxide, and the results of the tests with flasks stopped with cotton plugs and of those with flasks without cotton plugs were identical. Only if the plugs are flamed, it is necessary to wait about 6 hours before joining the upper flask with the lower. During the first few hours after flaming, the cotton continues to liberate carbon dioxide or some volatile organic acids which lower the titre of the barium hydroxide in the flask. Besides cultures in conic flasks, it is possible also to use cultures in test-tubes, the latter in this case are fitted into the cork with which the neck of the lower flask is sealed. In the case of cultures in liquid media, the upper conic flask must be exchanged for a round one, the neck of which is bent at a right angle (Fig. 2). The open end of this curvature is introduced into the lower flask and fixed there as described above, or else it is preliminarily passed through a cork with which afterwards the lower flask is stopped. In some cases it is useful for the sake of better aeration to make that part of the flask, in which is situated the liquid, flat.

As a matter of fact, the method proposed is sufficiently elastic and allows numerous variation. Thus, besides the determination of the liberated carbon dioxide, the apparatus permits to determine simultaneously the amount of oxygen consumed. For this purpose a manometer in the form of a V-shaped tube filled with mercury is affixed to the offset tube. In the same apparatus tests can also be arranged without oxygen; in this case the flasks are filled with inert gas—nitrogen or hydrogen.

Laboratory for Botany and Microbiology.
Leningrad State Roentgenological, Radiological and Cancerological Institute.

Received
25. III. 1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 5

PALEOZOOLOGY

A NEW *PROTELYTROPTERON* FROM THE PERMIAN OF THE URALS

By B. B. ROHDENDORF

(Communicated by A. A. Borissiak, Member of the Academy, 23.III.1939)

The present note describes a member of the peculiar Permian order *Protelytroptera* from the Kungurian deposits along the Sylva river; until now this group of insects has been known only from the Lower Permian deposits of North America (Kansas).

The occurrence of a representative of *Protelytroptera* in the Permian deposits on the Sylva river near the village of Tshekarda, a Kungurian age

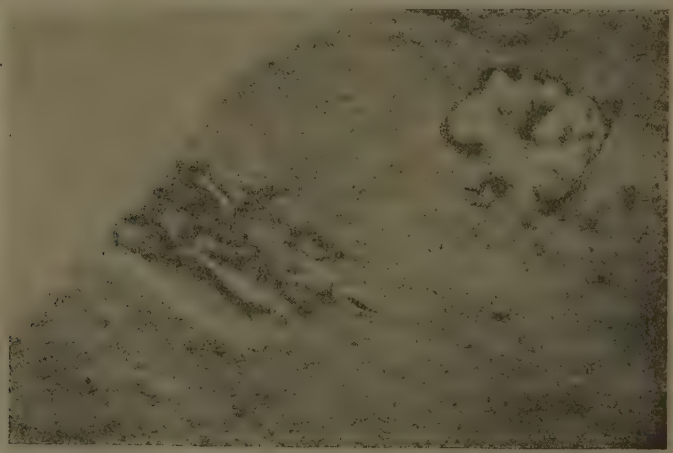


Fig. 1. *Uralelytron martynovi* gen. sp. n. Photograph from the impression № 72/168. × 5.

being ascribed to these deposits (Martynov, 1938), is of great interest. The first fact to be noted is that it is one more proof in favour of the relations of the Tshekarda fauna with that of Elmo, in Kansas, which has been up to now considered as undoubtedly Lower Permian. A study of the Tshekarda fauna conducted at present reveals the presence therein of a large number of archaic forms, pointing to its distinct relations to the Upper Carboniferous fauna. These discoveries make extremely significant the close

sesemblance with the Lower Permian fauna of Kansas. It is therefore posrible that the fauna of Tshekarda will be further on proved to be older than it has been assumed till now.

In general it should be stated that in making a comparison of the Permian fauna of the Soviet Union with that of North America and particularly with the Australian fauna, the purely geographical point should not be overlooked, i. e. the existence of well defined geographical provinces during the Permian time. I do not think it improbable that the Tshekarda and the Kansas fauna are almost completely synchronous.

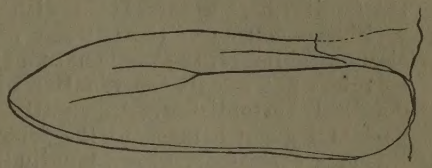


Fig. 2. Same, drawing from impression; × 10.

On the one hand, there are facts suggesting, as it would seem, a more ancient age of the fauna of Kansas; but on the other hand, there is an ample evidence to the reverse, pointing to a close relation of our Permian faunas to the Carboniferous one in Europe and North America.

Family *Protelytridae* (Till.) Carp. Genus *Uralelytron* gen. nov.
Genotype: *Uralelytron martynovi* sp. n.

T e g m e n: elongated in shape; length over three times the width; at the apex moderately pointed; strongly convex; its costal is in shape of a flat border, in the basal part of the wing with a sharp projection; posterior margin with a narrow, but distinctly pronounced border which is the widest in the distal part; surface covered with a distinct transverse rugosity, particularly well marked in the basal part. Venation much reduced; a short and delicate SC, rising from R(?) well marked, and R with RS arising from it more distally than the middle of the wing; other veins indistinct; possibly, their traces have remained, but are invisible owing to a superimposition of the basal part of the wing upon the tegmen at this point.

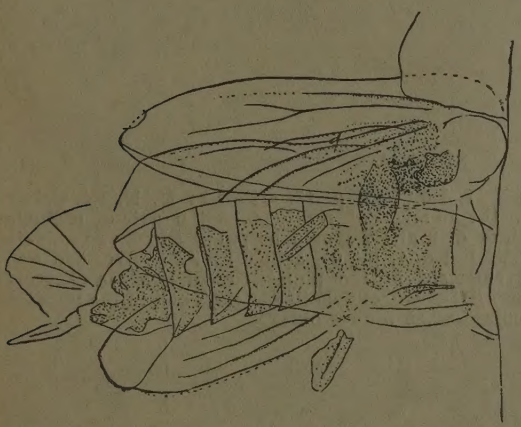


Fig. 3. Same, scheme of tegmen venation.

P o s t e r i o r w i n g: venation complete but hardly distinguishable, owing to the position of the wing. The peculiar fan of veins in the distal part distinct.

B o d y : head and part of thorax not preserved; margins of the abdomen seem to be parallel, consisting of eight visible segments; the first six segments rather short and approximately of equal length; the seventh segment narrower and considerably elongated; the eighth, very short and distally rounded. End of abdomen apparently carrying a pair of cerci, exceeding in length the three last segments of abdomen; their segmentation indistinct.

Legs not preserved.

A negative impression of the insect (Figs. 1—3). Tegmen somewhat shifted aside. Abdomen crushed and irregularly curved. Head and prothorax not preserved. Length of tegmen 6.5 mm, its greatest width 2.0 mm. Left bank of the Sylva river near the village of Tshekarda, 1938. E. V. Permjakova № 72/168.

The appurtenance of the genus described to the family *Protelytridae* arises no doubt. As to the relations of this genus with the other genera of the family, here we are faced with several difficulties. The strongly impoverished venation of the anterior wings cannot be considered as quite precisely established: the whole nature of the impression (negative, badly crumpled) does not permit the venation of the posterior half of the tegmen to be definitely established. Apparently the venation distinguishable in this part of tegmen belongs to the folded posterior wing; the tegmen itself in this area possessed very few longitudinal veins.

Thus, the genus *Uralelytron* gen. nov., having a structure of the radial system very much resembling that of the genus *Protelytron* Till., as well as a shape of tegmen common with the latter, is distinguished from this genus by a poorer venation in the system M, Cu and An. For the above reasons, it is as yet premature to draw from this the conclusion as to the greater specialization and relative youth of this genus, as compared to the genus *Protelytron* Handl. of Kansas.

Paleontological Institute.
Academy of Sciences of the USSR.
Moscow.

Received
27. III. 1939.

ERRATA

C. R. Acad. Sci. URSS, vol. XXI, No. 1—2, p. 16, article by M. N. Podaschewsky and A. M. Polonsky, fig. 5, the number of readings on the axis of ordinates should be reduced by 2, i. e. instead of 8 and 7 should be put 6 and 5 respectively.

C. R. Acad. Sci. URSS, vol. XXI, No. 4, p. 204, article by A. A. Woitkewitsch, subscription to fig. 3 read: «Development of the plumage as affected by thyroidectomy; right—5 days old duckling, left—control duckling 52 days old, in the middle—52 days old duckling thyroidectomized when 5 days old».

PRIX D'ABONNEMENT

Un an—36 livraisons . . . $ 15.00. Frs. fr. 250.00. £ 2.8.0
Six mois—18 livraisons . . $ 7.50. Frs. fr. 125.00. £ 1.4.0
(Séparément $ 0.50. Frs. fr. 8.00. £ 0.1.6 le numéro)

URSS—MEŽDUNARODNAJA KNIGA, 18, Kuznetskij Most, Moscou (Compte courant n° 263 Banque d'Etat de l'URSS, 12, Neglinnaja, Moscou).

U. S. A., NORTH and SOUTH AMERICA—BOOKNIGA INCORPORATED. 255 Fifth Avenue. New York, N. Y., USA.

ENGLAND —W. N. SMITH & son, Ltd. Strand House, Portugal Str. London W. C. 2.
—COLLET'S Foreign Department. 31 Gerrard Street, London W. I.

FRANCE —MESSAGERIES HACHETTE. Service Abonnements. 111, Rue Réaumur. Paris, 2e.

SUISSE —Buchhandlung STAUFFACHER (Genossenschaft). Hauptpost-Fach 43. Zürich.

ESPANA —«DISTRIBUIDORA DE PUBLICACIONES». Diputacion, 260. Barcelona.

Č. S. R. —MELANTRICH, Akc. Spol. Knihkupectvi — odd. slovanských knih. Václavské nám 42. Praha II. (Účet pošt. spoř. 20. 208).

BELGIQUE —OBLA (Office Belge Littéraire et Artistique). 6/8/10, rue Duquesnoy. Bruxelles.

LIETUVA —«MOKSLAS». Knygynas. Laisves Aleja, 46. Kaunas.
—SPAUDOS FONDAS. Laisves Al. 62. Kaunas.

TURQUIE —LIBRAIRIE HACHETTE. Succursale de Turquie. 469, Avenue de l'Indépendance. Boîte Postale Pera 2219. Istanbul.

DEUTSCHLAND —KOEHLER & VOLCKMAR, A/G. Russland-Abteilung. Postfach 173. Leipzig C. I.

POLSKA —GEBEHNER & WOLFF. Filja I. Krakowskie Przedm. 15, Warszawa.
—ROSSICA. Chmielna 5. Warszawa.

PALESTINE —SAFRAN CO. Jaffa Rd. 36. P. O. B. 223. Haifa.

LATVIJA —VALTERS un RAPA Akc. Sab. Teatra iela II. Riga (Pasta tek. rek. № 675).
K/s «LAIKMETS». Terbatas iela 37. Riga.

SVERIGE —C. E. FRITZE's Kgl. Hofbuchhandlung. Fredsgatan 2, Stockholm.
—A. B. WENNERCRENS WILLIAMS Box 657, Stockholm I.

AUSTRALIA —MODERN PUBLISHERS Ltd., 124 Oxford Street. Sydney.

BULGARIA —T. F. TSCHIPEFF. Buchhandlung Boulevard Dondoukoff 20/22 Sofia. (Postscheckkonto-Sofia 540).

NORGE —JOHAN GRUNDT TANUM. Tidligere Aschehougs Boghandel. Karl Johans gt. 43. Oslo.

ROMANIA —Biroul Central de Presa «LUVRU». Strada Oituz, 14, Bucuresti I.

EESTI —TÖÖKÖÖL, Pärnu mnt. 28, Tallinn.

DÄNEMARK —S. U. V. s Boghandel. Frederiksbarggade 42¹. Kopenhagen. K.

Les «Comptes Rendus», largement expédiés à l'étranger dans les centres scientifiques les plus importants et dans les bureaux de nombreuses revues, sont le seul organe scientifique en URSS, donnant tous les dix jours un compte rendu des plus récentes acquisitions de la science soviétique.

Prix *S* 0.50.